

Empirical Algorithms to Restore a Complete Set of Inherent Optical Properties of Seawater Using Any Two of These Properties

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RÉSUMÉ

Le présent travail fait l'analyse de l'information expérimentale et *in situ* mise au point par différents chercheurs et disponible, et la présente sous forme d'algorithmes simples pour permettre de reconstituer l'ensemble complet des propriétés optiques inhérentes de l'eau de mer. Les algorithmes suivants sont présentés : extraction de la fonction moyenne du cosinus de la distribution de la lumière dans la mer au moyen de l'albédo de diffusion simple; extraction de la probabilité de rétrodiffusion au moyen de l'albédo de diffusion simple; extraction de la fonction moyenne du cosinus du rayonnement descendant et ascendant de la distribution de la lumière dans la mer au moyen de l'albédo de diffusion simple; extraction du coefficient de réflexion diffuse au moyen de l'albédo de diffusion simple; extraction de la fonction de phase de diffusion de la lumière au moyen du coefficient de diffusion et de l'albédo de diffusion simple.

SUMMARY

This work analyzes available experimental and *in situ* information obtained by different investigators and presents it in a form of simple algorithms to restore the whole set of seawater inherent optical properties. The following algorithms are presented: retrieval of the total average cosine of the light distribution in the sea through the single scattering albedo; retrieval of backscattering probability through the single scattering albedo; retrieval of downward and upward average cosines of the light distribution in the sea through the single scattering albedo; retrieval of the diffuse reflection coefficient through the single scattering albedo; retrieval of the light scattering phase function through the scattering coefficient and the single scattering albedo.

Keywords

Ocean optics, inherent optical properties, scattering, absorption, seawater.

INTRODUCTION

For many practical applications, it is necessary to estimate the whole set of inherent optical properties (IOP) of seawater using limited information that includes only absorption and scattering coefficients. A complete set of inherent optical properties of seawater consists of absorption and angular scattering coefficients. Due to financial and/or technological limitations, the majority of *in situ* optical measurements does not include a complete set of hydro-optical properties. To use these incomplete data to estimate visibility parameters of seawater and calibrate atmospheric correction algorithms, it is necessary to have some means to estimate absent inherent optical properties through the measured ones.

The algorithms presented here are based on information obtained from experimental measurements of inherent optical properties and radiative transport theory. The optical data are processed in order to create easy engineering algorithms to retrieve a complete set of optical properties using an incomplete set of input data. The algorithms are derived from results of laboratory and *in situ* measurements by Timofeyeva (1971, 1972) and *in situ* measurements by Petzold (1972). The publications of Timofeyeva and Petzold are considered classical in ocean optics for two reasons: 1) they are unique because they consist of complete sets of hydro-optical measurements, and 2) they contain results of investigations in a tabular form. The set of algorithms proposed below allows us to predict all inherent optical properties, including phase function of scattering, using values of any pair of the following optical properties: absorption coefficient a , scattering coefficient b , beam attenuation coefficient, $c=a+b$ and single scattering albedo $\omega_0=b/c$.

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BACKSCATTERING COEFFICIENT AND AVERAGE COSINE

The backscattering coefficient of light in seawater is defined as

$$b_B = Bb, \quad (1)$$

where b is the probability of backscattering,

$$B = 0.5 \int_{\pi/2}^{\pi} p(\gamma) \sin \gamma d\gamma, \quad 0.5 \int_0^{\pi} p(\gamma) \sin \gamma d\gamma = 1, \quad (2)$$

here $p(\gamma)$ is the phase function of light scattering, and γ is the scattering angle in radians.

Downward $\bar{\mu}_d$, upward $\bar{\mu}_u$, and total $\bar{\mu}$ average cosines over the angular radiance distribution of light $L(\mu, \varphi)$ are defined according to the following equations:

$$\bar{\mu}_d = \int_0^{2\pi} d\varphi \int_0^1 L(\mu, \varphi) \mu d\mu / \int_0^{2\pi} d\varphi \int_0^1 L(\mu, \varphi) d\mu, \quad (3)$$

$$\bar{\mu}_u = - \int_0^{2\pi} d\varphi \int_{-1}^0 L(\mu, \varphi) \mu d\mu / \int_0^{2\pi} d\varphi \int_{-1}^0 L(\mu, \varphi) d\mu, \quad (4)$$

$$\bar{\mu} = \int_0^{2\pi} d\varphi \int_{-1}^1 L(\mu, \varphi) \mu d\mu / \int_0^{2\pi} d\varphi \int_{-1}^1 L(\mu, \varphi) d\mu, \quad (5)$$

here $\mu = \cos\theta$, θ , and φ are, respectively, polar and azimuth angles. In the general case, all average cosines depend on inherent optical properties [single scattering albedo ω_0 and phase function of scattering $p(\gamma)$], and conditions of illumination just below the sea surface. In an asymptotic diffuse regime, we can neglect spatial dependence of average cosines, because $L(z, \mu, \varphi) = L_\infty(\mu, \varphi) \exp(-az / \bar{\mu})$, where z is a distance from the sea surface. In an asymptotic regime average cosines depend only on inherent optical properties of seawater. According to Flateau *et al.*, (1999), due to a presence of air bubbles near the sea surface, an asymptotic regime is established very rapidly. In this regime, dependence of light fields on the scattering phase function is mostly reduced to a dependence on backscattering probability B (Gordon, 1993). According to Haltrin (1998a), and Haltrin and Weidemann (1996) the total average cosine $\bar{\mu}$ can be expressed as follows:

$$\bar{\mu} = \sqrt{\frac{1-g}{1+2g+\sqrt{g(4+5g)}}} \equiv \sqrt{\frac{1+2g-\sqrt{g(4+5g)}}{1+g}}, \quad (6)$$

or

$$\bar{\mu} = 0.5918 - 0.7937 g^{1/3} + 4.835 g^{2/3} - 22.815 g + 42.6859 g^{4/3} - 35.8945 g^{5/3} + 11.3905 g^2 + 0.4082 \sqrt{1-g}, \quad (6a)$$

here g is a Gordon's parameter defined as:

$$g = \frac{b_B}{a+b_B} \equiv \frac{B\omega_0}{1-\omega_0+B\omega_0}. \quad (7)$$

According to Equations (6) and (7) the total average cosine depends on two independent optical properties, ω_0 and B (or a and b_B). But under natural maritime conditions, some kind of biological equilibrium is usually established (Haltrin, 1999b). This equilibrium explains significant correlation between inherent optical properties (Timofeyeva, 1971; Petzold, 1972; Efimenko and Pelerin, 1975; Shannon, 1975; Haltrin, 1985, 1998a; Aas *et al.*, 1997).

It is possible to present experimental data by Timofeyeva (1971) as the following regression between the total average cosine $\bar{\mu}$ and the single scattering albedo ω_0 :

$$\bar{\mu} = y \{2.6178398 + y[-4.602418 + y(9.00406 + y\{-14.59994 + y[14.83909 + y(-8.117954 + 1.8593222y)]\})\}], \quad r^2 \cong 0.99. \quad (8)$$

$$y = \sqrt{1-\omega_0} \equiv \sqrt{a/c} \equiv \sqrt{1-b/c}.$$

In reverting Equation (6) it is possible to express Gordon's parameter through the total average cosine:

$$g = \frac{(1-\bar{\mu}^2)^2}{1+\bar{\mu}^2(4-\bar{\mu}^2)}. \quad (9)$$

In reverting Equation (7) in respect to B and using Equation (9), we have the following formula that connects backscattering probability with average cosine and single scattering albedo:

$$B = \frac{(1-\omega_0)g}{\omega_0(1-g)} \equiv \frac{(1-\omega_0)(1-\bar{\mu}^2)^2}{2\omega_0\bar{\mu}^2(3-\bar{\mu}^2)}. \quad (10)$$

Using Equation (8) with Equations (9) and (10) we can express Gordon's parameter, g , and backscattering probability, B , through one parameter, single scattering albedo ω_0 .

RETRIEVAL OF DOWNWARD AND UPWARD MEAN COSINES

Using experimental data collected by Timofeyeva (1972), which is supported by independent measurements by Efimenko and Pelerin (1975), we found the following expressions that connect downward and upward average cosines with the total average cosine $\bar{\mu}$:

$$\bar{\mu}_d = \frac{1-\bar{\mu}(1-\bar{\mu}^2)\{0.0326+\bar{\mu}^2[0.1661+\bar{\mu}^2(0.7785+0.0228\bar{\mu}^2)]\}}{2-\bar{\mu}}, \quad (11)$$

$$\bar{\mu}_u = \frac{1-0.987\bar{\mu}(1-\bar{\mu}^2)\exp\left\{\bar{\mu}^2\{8.4423+[-15.6605+\bar{\mu}^2(21.882-11.2257\bar{\mu}^2)]\}\right\}}{2-\bar{\mu}}. \quad (12)$$

Figure 1 shows regressional and experimental dependencies of average cosines as a function of Gordon's parameter [connected with $\bar{\mu}$ through Equation (9)]. Equations (8), (11)-(12) express all three average cosines $\bar{\mu}_d$, $\bar{\mu}_u$, and $\bar{\mu}$, through the value of a single scattering albedo ω_0 .

RETRIEVAL OF DIFFUSE REFLECTION AND DIFFUSE ATTENUATION COEFFICIENTS

The diffuse reflection coefficient, R_∞ , is determined by the ratio of upward to downward irradiances, or

$$R_\infty = - \frac{\int_0^{2\pi} d\varphi \int_{-1}^0 L(\mu, \varphi) \mu d\mu}{\int_0^{2\pi} d\varphi \int_0^1 L(\tau, \mu, \varphi) \mu d\mu} \quad (13)$$

Applying Equations (3-5) to Equation (13), it can be rewritten as:

$$R_\infty = \frac{1 - \bar{\mu} / \bar{\mu}_d}{1 + \bar{\mu} / \bar{\mu}_u} \equiv \frac{\bar{\mu}_u}{\bar{\mu}_d} \cdot \frac{\bar{\mu}_d - \bar{\mu}}{\bar{\mu}_u + \bar{\mu}} \quad (14)$$

Equation (14) connects R_∞ with average cosines $\bar{\mu}$, $\bar{\mu}_d$ and $\bar{\mu}_u$. Using Equations (8) and (11)-(12) R_∞ can be expressed through the single scattering albedo ω_0 . In more general cases Equations (6), (11)-(12) allow us to compute R_∞ as a function of g . **Figure 2** shows modelled and experimental dependencies of the diffuse reflection coefficient, R_∞ , and ratio $k=R_\infty/g$ on Gordon's parameter, g . From **Figure 2** it is clear that the simple equations of type $R_\infty = kg \equiv kb_B/(a+b_B)$, or $R_\infty = kg/(1-g) \equiv kb_B/a$, with a function $k=k(g)$ replaced by a constant, can be used only when $g \ll 1$ or $b_B \ll a$ [for example, in the case of Morel and Prieur, (1977), $k=1/3$]. For larger values of Gordon's parameter ($g \geq 0.1$) the substitution of function $k(g)$ by a constant is incorrect, and for the case of seawater we should use Equation (14) with Equations (6), (11)-(12) or Equation (64) in Haltrin (1998a). An example of restored values of R_∞ as a function of absorption and scattering coefficients is shown in **Table 1**.

The asymptotic diffuse attenuation coefficient of light \bar{k} is determined by Gershun's equation:

$$\bar{k} = \frac{a}{\bar{\mu}} \equiv c \frac{1 - \omega_0}{\bar{\mu}}, \quad (15)$$

with $\bar{\mu}$ defined by Equation (8).

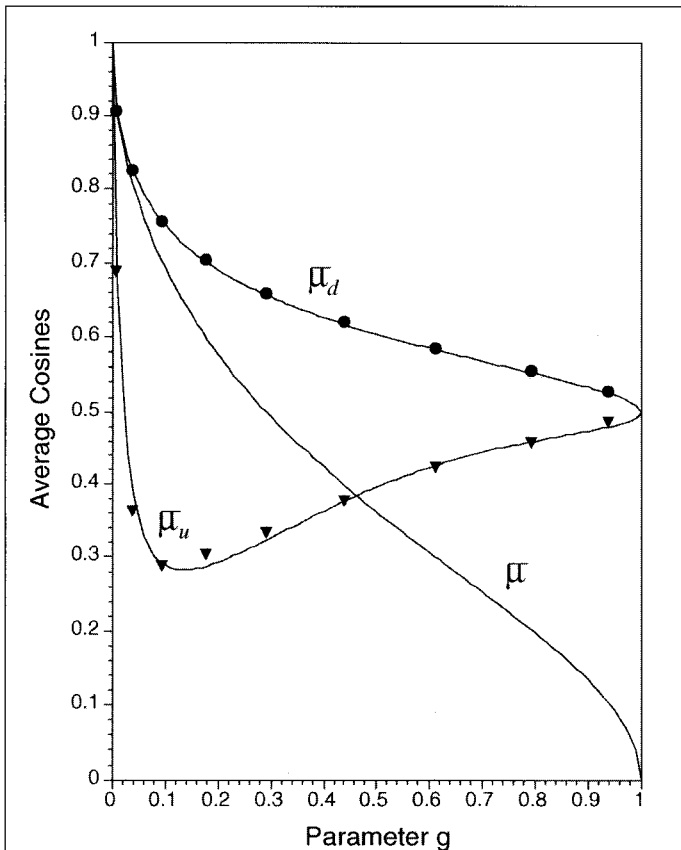


Figure 1. Average cosines of light propagating in seawater as a function of Gordon's parameter g . The lines show the values computed using this model, while the symbols correspond to the experimental data by Timofeyeva (1972).

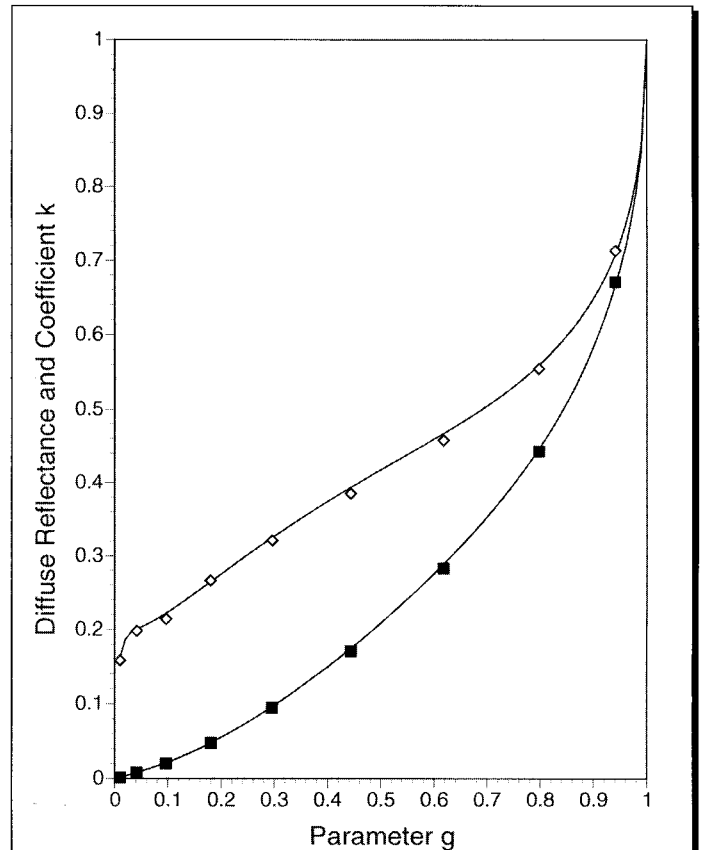


Figure 2. Diffuse reflectance coefficient R_∞ and ratio $k=R_\infty/g$ as a function of Gordon's parameter g . The lines correspond to calculations with this model, and the symbols represent experimental values by Timofeyeva (1971).

Table 1.
Values of diffuse reflectance coefficient, R_∞ , in % for different pairs of absorption and scattering coefficients (in m^{-1}).

a/b	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000
0.010	15.338	25.445	31.973	36.710	40.391	43.380	45.883	48.025	49.892	51.540
0.020	7.749	15.338	21.033	25.445	29.006	31.973	34.507	36.710	38.655	40.391
0.050	2.527	6.000	9.436	12.559	15.338	17.808	20.015	22.001	23.802	25.445
0.100	1.031	2.527	4.231	6.000	7.749	9.436	11.041	12.559	13.990	15.338
0.200	0.395	1.031	1.742	2.527	3.364	4.231	5.114	6.000	6.880	7.749
0.300	0.205	0.600	1.031	1.496	1.997	2.527	3.080	3.650	4.231	4.819
0.500	0.078	0.279	0.517	0.768	1.031	1.306	1.594	1.894	2.205	2.527
1.000	0.017	0.078	0.170	0.279	0.395	0.517	0.641	0.768	0.898	1.031
1.500	0.006	0.033	0.078	0.137	0.205	0.279	0.356	0.436	0.517	0.600
2.000	0.003	0.017	0.043	0.078	0.121	0.170	0.223	0.279	0.336	0.395

The downward and upward depth dependent diffuse attenuation coefficients \bar{k}_d and \bar{k}_u may be determined by Equations (58)-(60) in Haltrin (1998a) or Equations (42)-(43) in Haltrin (1998b). An example of restored values of \bar{k} as a function of absorption and scattering coefficients is shown in Table 2.

$$p(\gamma) = \frac{4\pi}{l_0 b} \exp \left[q \left(1 + \sum_{n=1}^5 k_n \gamma^n \right) \right], \quad (16)$$

here γ is the scattering angle in radians, b is in m^{-1} , $l_0=1m$. Coefficients q and $k_n(n=1, \dots, 5)$ are given by the following equations:

RETRIEVAL OF SEAWATER LIGHT SCATTERING PHASE FUNCTION

By studying fifteen experimental values of the scattering phase function, $p(\gamma)$, published by Petzold (1972) with the corresponding values of a , b and B , we obtained the following regressions that connect $p(\gamma)$ with b and ω_0 :

$$\left. \begin{aligned} q &= 2.598 + 5.932 \sqrt{l_0 b} (2.992 + l_0 b) - 16.722 l_0 b, & r^2 &= 0.996, \\ k_1 &= 5.2077 \omega_0 - 8.9924, & r^2 &= 0.925, \\ k_2 &= 17.59 - 10.886 \omega_0, & r^2 &= 0.897, \\ k_3 &= 13.098 \omega_0 - 19.863, & r^2 &= 0.893, \\ k_4 &= 10.636 - 7.386 \omega_0, & r^2 &= 0.887, \\ k_5 &= 1.515 \omega_0 - 2.087, & r^2 &= 0.870. \end{aligned} \right\} \dots 17)$$

Table 2.
Values of diffuse attenuation coefficient $\bar{k} = a/\bar{\mu}$ (in m^{-1}) for different pairs of absorption and scattering coefficients (in m^{-1}).

a/b	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.010	0.0239	0.0309	0.0363	0.0409	0.0449	0.0486	0.0519	0.0551	0.0580	0.0608
0.020	0.0379	0.0477	0.0554	0.0618	0.0675	0.0727	0.0774	0.0818	0.0860	0.0899
0.050	0.0737	0.0885	0.1003	0.1104	0.1193	0.1274	0.1349	0.1418	0.1484	0.1546
0.100	0.1284	0.1473	0.1631	0.1769	0.1893	0.2006	0.2110	0.2207	0.2299	0.2386
0.200	0.2336	0.2568	0.2767	0.2947	0.3111	0.3263	0.3405	0.3538	0.3665	0.3785
0.300	0.3368	0.3628	0.3852	0.4055	0.4243	0.4420	0.4586	0.4744	0.4894	0.5037
0.500	0.5408	0.5708	0.5966	0.6200	0.6419	0.6626	0.6823	0.7011	0.7192	0.7366
1.000	1.0454	1.0816	1.1131	1.1416	1.1681	1.1931	1.2170	1.2401	1.2623	1.2838
1.500	1.5474	1.5873	1.6224	1.6545	1.6842	1.7123	1.7391	1.7648	1.7897	1.8138
2.000	2.0486	2.0907	2.1286	2.1632	2.1956	2.2261	2.2552	2.2831	2.3100	2.3361

The phase function, $p(\gamma)$, given by Equations (16)-(17) can be used as a basis for an empirical model of the light scattering phase function in seawater that depends on absorption and scattering coefficients. Single-scattering albedo, $\omega_0=b/c$, used here varies from 0.09 to 0.96.

Figure 3 displays all fifteen Petzold phase functions plotted with the phase functions computed with this algorithm through the pairs of a s and b s taken from Petzold (1972). The overall agreement between computed and experimental values lies in the range of 10%.

COMPARISON OF RESTORED VALUES OF IOP WITH EXPERIMENTAL DATA NOT USED TO DERIVE THIS ALGORITHM

In situ measurements of the complete set of inherent optical properties (i.e., a , b and phase function) that are released for public use are limited to the report by Petzold (1972) and unpublished data by Man'kovsky (1995) reported in Haltrin (1997). The Petzold data were used to derive this algorithm. To compare results of restoration with independent measurements, we use the following three oceanic measurements made by Man'kovsky (1995).

Measurement # 1: Indian Ocean, tropical zone, March 1975, phase function # 3, depth of measurements is 7 m.

Measurement # 2: Atlantic Ocean, tropical zone, September 1986, phase function # 4, depth of measurements is 3 m.

Measurement # 3: Atlantic Ocean, tropical zone, September 1986, phase function # 5, depth of measurements is 3 m.

The values of all IOPs for these measurements, excluding phase functions, are given in Table 3. The angular dependence of Man'kovsky phase functions are given in Haltrin (1997) in a form of empirical equations and may be obtained on request from the author (V. I. Man'kovsky). To restore the whole set of IOPs we used a program 'fiopexp' published in Haltrin (1999a) and pairs of input values of b and c from Table 3. This program can be easily modified to accept any pair of independent values of IOPs: (a, b) , (a, c) , (a, ω_0) , (b, ω_0) , (c, ω_0) , etc.

Values of b_B restored with this algorithm are given in Table 3, and comparison of restored and measured values of scattering phase functions is presented in Figure 4.

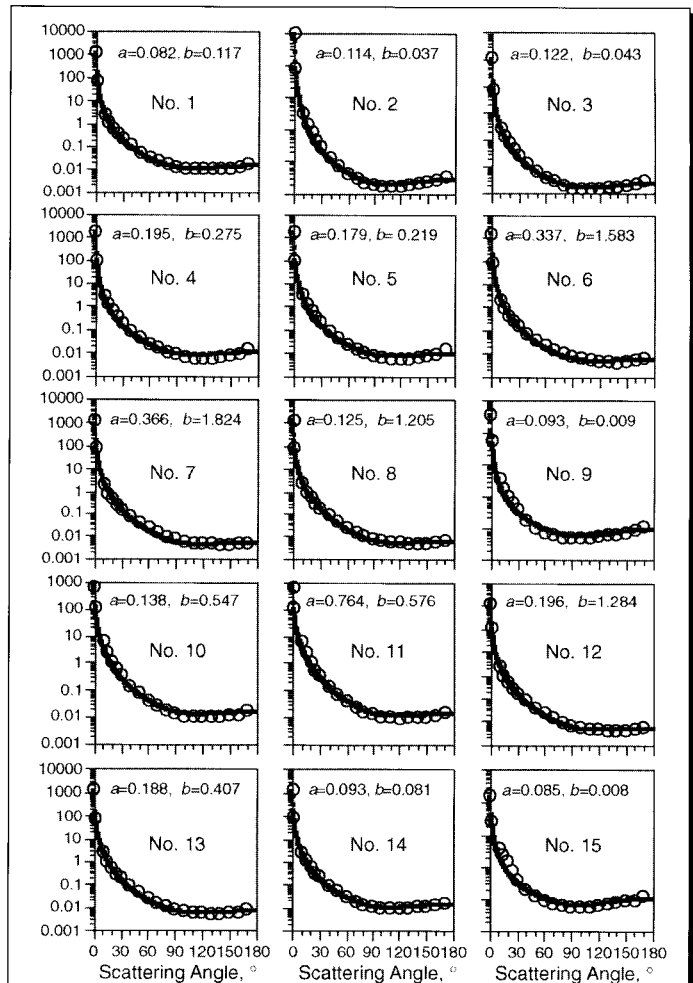
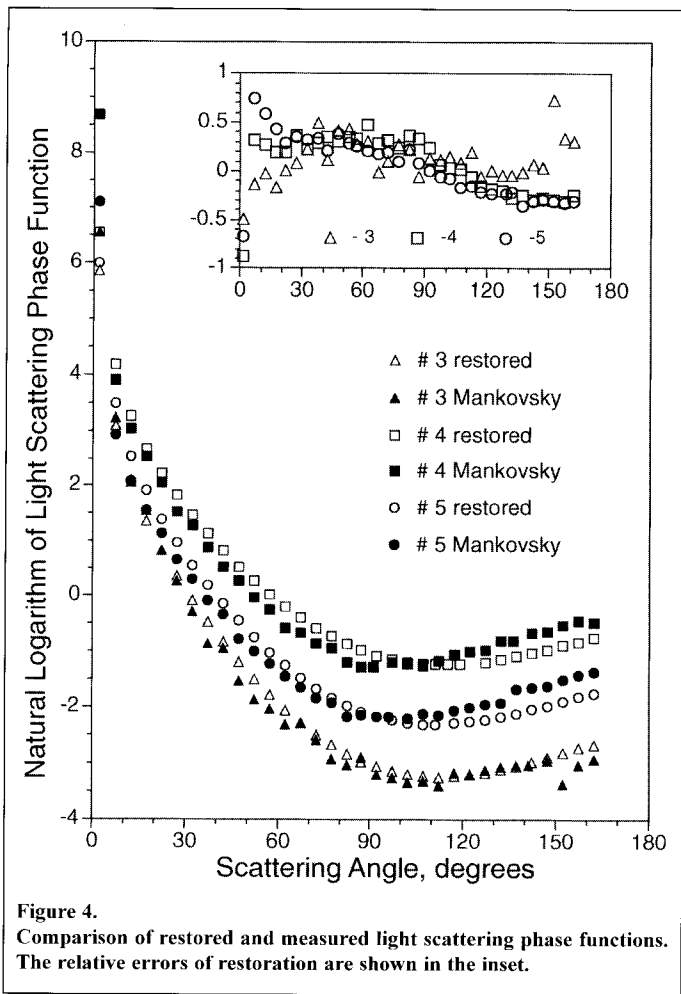


Figure 3. Phase functions of light scattering in seawater $p(\gamma)$ computed with this algorithm (dark symbols) and experimental Petzold phase functions (open circles). The pairs of a and b used for the input are taken from Petzold (1972).

The overall error of phase function restoration in the angular range from 20 to 150 degrees lies in the range of $\pm 50\%$ (see Figure 4). The error of restoration of backscattering coefficient b_B is less than 50% for all three Man'kovsky phase functions (see Table 3). Provided that the phase function portion of the algorithm is based on measurements made in the Pacific Ocean, this level of restoration should be regarded as satisfactory. Further improvement of the algorithm may be accomplished only when a more extensive database of complete sets of IOPs will be available for analysis.

Table 3. Restored values of backscattering coefficient $b_{B|restored}$ with corresponding IOPs.

#	Location	a, m^{-1}	b, m^{-1}	c, m^{-1}	ω_0	b_B, m^{-1}	$b_{B restored}, m^{-1}$	error
1	Indian Ocean	0.1197	0.1566	0.2763	0.5667	0.002796	0.003681	32%
2	Atlantic Ocean	0.0553	0.1059	0.1612	0.6571	0.002254	0.002787	24%
3	Atlantic Ocean	0.0806	0.0622	0.1428	0.4355	0.002303	0.001211	47%



CONCLUSION

A set of equations presented in this paper allows us to calculate the whole set of inherent optical properties from any pair of the following input parameters: absorption coefficient a , scattering coefficient b , beam attenuation coefficient $c=a+b$, and single scattering albedo ω_0 . This model is implemented as a FORTRAN 77 code (published in Haltrin, 1999a). The output of this code contains backscattering coefficient b_B , probability of backscattering B , total, downward and upward average cosines, diffuse reflection and attenuation coefficients, and tables of scattering phase function for chosen angles. This model may be used to estimate unmeasured optical properties from the results of incomplete *in situ* experiments [for example, to estimate scattering phase functions and backscattering coefficients using results of AC-9 measurements (WETLabs Inc., 1994) of a and c].

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