EFFECTS OF RAMAN SCATTERING AND FLUORESCENCE ON APPARENT OPTICAL PROPERTIES OF SEA WATER

Self-Consistent Solutions to the Equation of Transfer with Raman Scattering and Chlorophyll and Yellow Substance Fluorescence in Sea Water: Model and Results

by

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1. Introduction

Although the literature abounds with two stream approximations (see Refs [1, 2, 3] for a thorough listing of most of the relevant papers), we will present here an extension of a new two stream method [1] which will allow for inelastic processes such as Raman scattering and fluorescence. What sets this method apart from all other two stream approximations is the fact that it allows the option of actually adjusting certain parameters to fit some experimental data obtained from either real marine waters such as the Mediterranean sea, Black sea, Atlantic ocean [4], and Indian ocean [5] or emulated scattering and absorbing media [4]. This method also has the versatility to cover a *complete* range of inherent oceanic parameters ranging from the very clear to the most turbid (with concentration of chlorophyll up to 3 mg/m^3 which corresponds to the most productive waters of the open ocean). It allows for both molecular and hydrosol scattering, as well as both chlorophyll and *yellow substance* absorption. At present the model only allows for a homogeneous ocean with a flat surface; however, the extension to an inhomogeneous ocean can be obtained with relatively little work but it will increase the time per calculation by about a factor of ten. It can also be extended to include a stochastic interface but this will take considerably more work. The input into this model can be quite arbitrary and allows for a realistic solar spectral input which is a *sine qua non* for inelastic scattering processes.

The program was written in Fortner Research LLC LS-FORTRAN 77 to run on an Apple 68k- and Power Macintoshes and its compiled version runs in about four minutes on Macintosh IIx and in about five seconds on Power Macintosh 8100/80av and covers a spectral range from 200–700 *nm* and depth range between 0–200 *m*. It can easily be ported to run on IBM compatible PCs and VAXes. The electronic copy of this report in PostScript format and the program itself (available for 68k Macintosh, Power Macintosh, Sun and SGI UNIX machines) is stored in public anonymous account at <indyvih.nrlssc.navy.mil>.

We will present a brief derivation of the relevant equations and then show some sample results.

2. Nomenclature

2.1. Notations in equations and tables

a	absorption coefficient (in *m-1*)
$b = b^E + b^R$	total scattering coefficient (in *m-1*)
$b_B = b^E B$	*elastic* backscattering coefficient
b^E	*elastic* scattering coefficient (in *m-1*)
b^R	Raman scattering coefficient (in *m-1*)
$B = 0.5\int_{-1}^{0} p(\mu)d\mu$	backscattering probability
$c^E = a + b^E$	elastic extinction coefficient (in *m-1*)
$c = a + b$	extinction coefficient (in *m-1*)
d	euphotic zone depth without effects of inelastic emission
$d^E \equiv d(b^R = 0)$	*elastic* euphotic zone depth
E_d^F	fluorescent contribution to downward irradiance
E_u^F	fluorescent contribution to upward irradiance
E_d^R	Raman scattering contribution to downward irradiance
E_u^R	Raman scattering contribution to upward irradiance
$E_d^{CY} = E_d^C + E_d^Y$	*inelastic* scattering contribution to downward irradiance
$E_u^{CY} = E_u^C + E_u^Y$	*inelastic* scattering contribution to upward irradiance
E_d^q	downward irradiance of *direct* light
E_d^s	downward irradiance of elastically *scattered* light
$E_d^E = E_d^q + E_d^s$	downward irradiance without effects of inelastic emission
E_u^s	upward irradiance of elastically *scattered* light
E_{0d}^s	downward scalar irradiance of elastically *scattered* light
E_{0u}^s	upward scalar irradiance of elastically *scattered* light
$f^F(\lambda', \lambda)$	fluorescence radiance emittance function
$f^R(\lambda', \lambda)$	Raman radiance emittance function
k_d	total downward irradiance attenuation coefficient
k_u	total upward irradiance attenuation coefficient
k_d^E	*elastic* downward irradiance attenuation coefficient
k_u^E	*elastic* upward irradiance attenuation coefficient
k_0	attenuation coefficient of upward *scattered* light (without effects of inelastic emission)
$k_0^E \equiv k_0(b^R = 0)$	attenuation coefficient of upward elastically *scattered* light
k_∞	attenuation coefficient of downward *scattered* light (without effects of inelastic emission)
$k_\infty^E \equiv k_\infty(b^R = 0)$	attenuation coefficient of downward elastically *scattered* light

$L^E(\lambda,\mathbf{r},\mathbf{n})$	*elastic* radiance
$L^F(\lambda,\mathbf{r},\mathbf{n})$	radiance for fluorescent contribution
$L^R(\lambda,\mathbf{r},\mathbf{n})$	radiance for Raman scattering contribution
$p(\mathbf{nn'})$	scattering phase function
$p_R(\mathbf{nn'})$	Rayleigh scattering phase function
$Q^E(\lambda,\mathbf{r},\mathbf{n})$	*elastic* scattering source function
R	diffuse reflectance of sea illuminated by direct sunlight (includes Raman scattering and fluorescent effects)
R_s	diffuse reflectance of sea illuminated by direct sunlight (without effects of inelastic emission)
$R_s^E \equiv R_s(b^R = 0)$	diffuse reflectance of sea illuminated by direct sunlight (without effects of inelastic scattering)
R_w	diffuse reflectance of infinitely optically deep sea illuminated by diffuse light (without effects of inelastic emission)
$R_w^E \equiv R_w(b^R = 0)$	diffuse reflectance of infinitely optically deep sea illuminated by diffuse light (without effects of inelastic scattering)
R_∞	diffuse reflectance in the asymptotic regime (without effects of inelastic emission)
$R_\infty^E \equiv R_\infty(b^R = 0)$	*elastic* diffuse reflectance in the asymptotic regime
$x = b_B/(a+b_B)$	Gordon's parameter
$\tilde{x} = b_B/(\tilde{a}+b_B)$	Gordon's parameter (without effects of inelastic scattering)
z	depth coordinate (in *m*)
$z_\oplus$	solar zenith angle
α	renormalized attenuation coefficient for scattered light without effects of inelastic emission (in m^{-1})
$\alpha^E = a + 2b_B$	renormalized *elastic* attenuation coefficient for scattered light (in m^{-1})
$\gamma = \cos^{-1}\mathbf{nn'}$	scattering angle
δR^R	Raman scattering corrections to R_s
δR^Y	yellow substance fluorescent corrections to R_s
δR^C	chlorophyll fluorescent corrections to R_s
$\delta R^I \ (I = R,Y,C)$	*inelastic* correction to R_s
$\delta R^F \ (F = Y,C)$	fluorescent correction to R_s
$\delta R^{CY} = \delta R^C + \delta R^Y$	total fluorescent corrections to R_s
λ	wavelength of light (in *nm*)
$\bar{\mu}$	mean cosine without effects of inelastic emission
$\bar{\mu}^E \equiv \bar{\mu}(b^R = 0)$	*elastic* mean cosine
$\bar{\mu}_d$	downward mean cosine without effects of inelastic emission
$\bar{\mu}_u$	upward mean cosine without effects of inelastic emission

θ	azimuth angle
$\tau = \alpha z$	renormalized optical depth
φ	polar angle
$\omega_0 = b^E / c^E$	*elastic* single scattering albedo
$\varpi_0 = b^E / c$	single scattering albedo

2.2. Notations in legends of Figures

AL = ω_0	elastic single scattering albedo
ALI = ϖ_0	inelastic single scattering albedo
CE = c^E	elastic extinction coefficient
CI = c	inelastic extinction coefficient
D = d^E	elastic euphotic zone depth
DI = d	inelastic euphotic zone depth
DRF = $\delta R^Y + \delta R^C$	fluorescent correction to R_s (RS)
DRI = $\delta R^R + \delta R^Y + \delta R^C$	inelastic correction to R_s
DRR = δR^R	Raman scattering correction to R_s
EFD = $E_d^Y + E_d^C$	downward irradiance for fluorescent contribution
EFU = $E_u^Y + E_u^C$	upward irradiance for fluorescent contribution
ERD = E_d^R	downward irradiance for Raman scattering contribution
ERU = E_u^R	upward irradiance for Raman scattering contribution
KBIA = αk_∞	asymptotic inelastic attenuation coefficient
KEED = k_d^E	total downward elastic irradiance attenuation coefficient
KEEU = k_u^E	total upward elastic irradiance attenuation coefficient
KEID = k_d	total downward inelastic irradiance attenuation coefficient
KEIU = k_u	total upward inelastic irradiance attenuation coefficient
RI = R	diffuse reflectance of sea illuminated by direct sunlight (includes Raman scattering and fluorescent effects)
RS = R_s	diffuse reflectance of sea illuminated by direct sunlight (without effects of inelastic emission)
X = x	*elastic* Gordon's parameter
XI = $\tilde{x}$	*inelastic* Gordon's parameter

3. Basic Equations

We start with the scalar transfer equation for spectral radiance $L(\lambda,\mathbf{r},\mathbf{n})$ [6, 7] which considers such inelastic effects as Raman scattering and fluorescence

$$[\mathbf{n}\nabla + c(\lambda)]\ L(\lambda,\mathbf{r},\mathbf{n}) \ = \ Q(\lambda,\mathbf{r},\mathbf{n}) \ , \tag{1}$$

$$Q(\lambda,\mathbf{r},\mathbf{n}) \ = \ Q^E(\lambda,\mathbf{r},\mathbf{n}) + \sum_I Q^I(\lambda,\mathbf{r},\mathbf{n}) + Q^{HO}(\lambda,\mathbf{r},\mathbf{n}) \ , \tag{2}$$

where $Q^E(\lambda,\mathbf{r},\mathbf{n})$ is the *elastic* scattering source function, $\sum_I Q^I(\lambda,\mathbf{r},\mathbf{n})$ is a sum of the *inelastic* source terms, $Q^{HO}(\lambda,\mathbf{r},\mathbf{n})$ describes higher orders of *inelastic* scattering. We represent the total radiance as

$$L(\lambda,\mathbf{r},\mathbf{n}) \ = \ L^E(\lambda,\mathbf{r},\mathbf{n}) + \sum_I L^I(\lambda,\mathbf{r},\mathbf{n}) \ ,$$

here $L^E(\lambda,\mathbf{r},\mathbf{n})$ is the *elastic* radiance, *i. e.* radiance in the absence of Raman and fluorescent effects, and $L^I(\lambda,\mathbf{r},\mathbf{n})$ is the *inelastic* radiance ($I = R, F$), $c(\lambda) = a(\lambda) + b(\lambda)$ is the total extinction coefficient, $a(\lambda)$ is the absorption coefficient, $b(\lambda) = b^E(\lambda) + b^R(\lambda)$ is the total scattering coefficient, $b^E(\lambda)$ is the elastic scattering coefficient by water molecules (Rayleigh) and hydrosol particles (Mie), $b^R(\lambda)$ is the inelastic Raman scattering coefficient.

The source terms can be represented by the following formulas

$$Q^E(\lambda,\mathbf{r},\mathbf{n}) \ = \ \frac{b^E(\lambda)}{4\pi}\int p(\mathbf{nn}')\ L^E(\lambda,\mathbf{r},\mathbf{n}')\ d\mathbf{n}' \ , \tag{3}$$

$$Q^I(\lambda,\mathbf{r},\mathbf{n}) = \frac{b^E(\lambda)}{4\pi}\int p(\mathbf{nn}') L^I(\lambda,\mathbf{r},\mathbf{n}') d\mathbf{n}' + \frac{1}{4\pi}\int d\lambda' \int \sigma^I(\lambda',\lambda,\mathbf{nn}') L^E(\lambda',\mathbf{r},\mathbf{n}') d\mathbf{n}', \tag{3a}$$

$$\sigma^F(\lambda',\lambda,\mathbf{nn}') \ \equiv \ \sigma^F(\lambda',\lambda) \ = \ b^F(\lambda') f^F(\lambda',\lambda) \ , \tag{3b}$$

$$\sigma^R(\lambda',\lambda,\mathbf{nn}') \ = \ b^R(\lambda) f^R(\lambda',\lambda)\, p_R(\mathbf{nn}'); \tag{3c}$$

$$b^F(\lambda) \ = \ \eta_F\, a_F^0(\lambda) F \ , \quad b^R(\lambda) \ = \ b_R^0 \left(\frac{400}{\lambda}\right)^4 ; \tag{3d}$$

$$\int f^I(\lambda',\lambda) d\lambda' \ = \ \int f^I(\lambda',\lambda) d\lambda \ = \ 1 \ , \quad p_R(\mathbf{nn}') \ = \ \frac{3}{4}[1 + (\mathbf{nn}')^2], \tag{3e}$$

here $f^R(\lambda',\lambda)$ and $f^F(\lambda',\lambda)$ are Raman and fluorescence radiance emittance functions respectively, and $p_R(\mathbf{nn}')$ is the Rayleigh scattering function, with $\gamma = \cos^{-1}\mathbf{nn}'$, the scattering angle. All phase functions in this report are normalized according to

$$\int p(\mathbf{nn}') d\mathbf{n}' \equiv \int_0^{2\pi} d\varphi \int_0^{\pi} p(\cos\gamma) \sin\theta\, d\theta = 4\pi \ .$$

If we denote as $\varphi_1 \otimes \varphi_2 \equiv \frac{1}{4\pi}\int d\lambda' \int d\mathbf{n}' \; \varphi_1(\lambda', \mathbf{n}', \ldots) \; \varphi_2(\lambda', \mathbf{n}', \ldots)$, then $Q^{HO} = \left(\sum_I \sigma^I\right) \otimes \left(\sum_I L^I\right)$. Actually this term is very small and we will neglect it in our calculations.

Introducing the operator

$$\hat{T}\varphi \equiv \left[\mathbf{n}\nabla + c - \frac{b^E}{4\pi}\int d\mathbf{n}' p(\mathbf{n}\mathbf{n}')\right]\varphi \;, \tag{4}$$

and neglecting the small term Q^{HO} we can rewrite (1) as

$$\hat{T}\, L^E = 0 \;, \tag{5}$$

$$\hat{T}\, L^I = \sigma^I \otimes L^E \;, \tag{6}$$

$$L = L^E + \sum_I L^I \;. \tag{7}$$

4. Approximate Solutions for Elastic Anisotropically–Scattering Part

In this section we obtain a solution to equation (5) for the irradiances which will be as simple as possible and yet have a precision of 5–10% and valid in the complete range of variability of inherent optical properties, *i. e.* from very clean to very turbid ocean waters. In the absence of Raman scattering and fluorescence the exact equation for the scalar radiance is

$$[\mathbf{n}\nabla + c] \;\; L^E(\mathbf{r}, \mathbf{n}) = \frac{b^E}{4\pi}\int d\mathbf{n}' p(\mathbf{n}\mathbf{n}') L^E(\mathbf{r}, \mathbf{n}') \;\;, \tag{8}$$

and since we are only considering elastic scattering, we have dropped the parameter λ.

In this section we will now use the approach in Ref. **1**, namely:

a) we will neglect horizontal inhomogeneities of the sea;

b) we will represent the highly anisotropic sea water volume scattering function in the form, $p(\mathbf{n}\mathbf{n}') = p_T(\mathbf{n}\mathbf{n}') + p_\Delta(\mathbf{n}\mathbf{n}')$, where $p_T(\mathbf{n}\mathbf{n}') = 2B + 2(1-2B)\delta(1-\mathbf{n}\mathbf{n}')$ is the *transport* phase function; $B = 0.5\int_{-1}^{0} p(\mu)\, d\mu$ is the backscattering probability $(\mu = \mathbf{n}\mathbf{n}')$; $\delta(x)$ is the Dirac delta-function, and $p_\Delta = p - p_T$.

Later we retain only the *transport* part of p and take into account the influence of p_Δ indirectly.

Let us introduce the renormalized optical depth $\tau = \alpha z$ where $\alpha = \tilde{a} + 2b_B$, which is the renormalized attenuation coefficient, $b_B = b^E B$ is the elastic backscattering coefficient, $\tilde{a} = c - b^E \equiv a + b^R$ and $\tilde{x} = b_B/(\tilde{a} + b_B)$, we then get

$$\left(\mu \frac{d}{d\tau} + 1\right) L^E(\tau,\mu,\varphi) = \frac{1}{2\pi}\left[\frac{\tilde{x}}{1+\tilde{x}} \int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu' L^E(\tau,\mu',\varphi') + \Delta(\tau,\mu,\varphi)\right], \quad (9)$$

$$\Delta(\tau,\mu,\varphi) = \frac{\varpi_0 \ (1-\tilde{x})}{2(1-\varpi_0)(1+\tilde{x})} \int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu' [p(\mu') - p_T(\mu')] L^E(\tau,\mu',\varphi') , \quad (10)$$

here $\varpi_0 = b^E/(\tilde{a} + b^E) = b^E/c$ is the single scattering albedo.

Let us represent the *elastic* radiance as a sum of *scattered* and *unscattered* light

$$L^E = L^s + L^q . \quad (11)$$

We can postulate that L^q should satisfy the equation

$$\left(\mu \frac{d}{d\tau} + 1\right) L^q(\tau,\mu,\varphi) = 0 , \quad (12)$$

with the boundary condition $L^q(0,\mu,\varphi) = L_0^q(\mu,\varphi), \ \mu > 0$.

The solution of (12) will be

$$L^q(\tau,\mu,\varphi) = L_0^q(\mu,\varphi) \ e^{-\tau/\mu} , \quad (13)$$

Then *scattered* radiance should satisfy the equation

$$\hat{T}_\Delta L^s = \frac{g(\tau)}{2\pi} , \quad (14)$$

where

$$\hat{T}_\Delta L^s \equiv \left(\mu \frac{d}{d\tau} + 1\right) L^s(\tau,\mu,\varphi) - \frac{1}{2\pi}\left[\frac{\tilde{x}}{1+\tilde{x}} \int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu' L^s(\tau,\mu',\varphi') + \Delta(\tau,\mu,\varphi)\right], \quad (15)$$

with the boundary conditions

$$L^s(0,\mu,\varphi)\Big|_{\mu>0} = 0, \qquad \lim_{\tau\to\infty} L^s(\tau,\mu,\varphi) = 0 , \quad (15a)$$

here

$$g(\tau) = \frac{\tilde{x}}{1+\tilde{x}} \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu \ L_0^q(\mu,\varphi) \ e^{-\tau/\mu} . \quad (16)$$

The function $\Delta(\tau,\mu,\varphi)$ vanishes in two cases: isotropic scattering $(B = 1/2)$, and highly anisotropic scattering $(B = 0)$.

Now we rewrite Eqn. (14) in terms of irradiances [8]:

$$E_d^s = \int_0^{2\pi} d\varphi \int_0^1 L^s(\mu,\varphi)\mu\, d\mu \ , \quad E_u^s = -\int_0^{2\pi} d\varphi \int_{-1}^0 L^s(\mu,\varphi)\mu\, d\mu \ , \qquad (17)$$

$$E_{0d}^s = \int_0^{2\pi} d\varphi \int_0^1 L^s(\mu,\varphi) d\mu \ , \quad E_{0u}^s = \int_0^{2\pi} d\varphi \int_{-1}^0 L^s(\mu,\varphi) d\mu , \qquad (18)$$

$$E_0^s = \int_0^{2\pi} d\varphi \int_{-1}^1 L^s(\mu,\varphi)\, d\mu = E_{0d}^s + E_{0u}^s \ . \qquad (19)$$

We also introduce mean cosines (see Ref. [8]):

$$\bar{\mu}_d = \frac{E_d^s}{E_{0d}^s} \ , \quad \bar{\mu}_u = \frac{E_u^s}{E_{0u}^s} \ , \quad \bar{\mu} = \frac{E_d^s - E_u^s}{E_0^s} = \frac{E_d^s - E_u^s}{E_{0d}^s + E_{0u}^s} \ . \qquad (20)$$

Applying the operators $\int_0^{2\pi} d\varphi \int_0^1 \mu\, d\mu \ldots$ and $-\int_0^{2\pi} d\varphi \int_{-1}^0 \mu\, d\mu \ldots$ to (14) and taking into account $\Delta(\tau,\mu,\varphi)$ indirectly by accepting the following dependencies [2, 4]: $\bar{\mu}_d = 1/(2-\bar{\mu})$, $\bar{\mu}_u = 1/(2+\bar{\mu})$, we get the system of equations:

$$\hat{L}_{ik}(\tau) E_k^s = e_i\, g(\tau) \ , \quad \hat{L}_{ik}(\tau) = \begin{Vmatrix} \frac{d}{d\tau} + q_- & -\tilde{x}\, q_+ \\ -\tilde{x}\, q_- & -\frac{d}{d\tau} + q_+ \end{Vmatrix} \qquad (21),$$

where $e_i = (1,\ 1)$, $q_\pm = (2 \pm \bar{\mu})/(1+\tilde{x})$. From here on we assume summation over repeated indices. System (21) has two eigenvalues $-k_\infty$ and k_0. Imposing the principle of self-consistency [1], *i. e.*, $k_\infty \equiv \frac{\tilde{a}}{\alpha\bar{\mu}} = \frac{1-\tilde{x}}{(1+\tilde{x})\bar{\mu}}$, we get the following equations for $\bar{\mu}$, k_∞ and k_0:

$$\bar{\mu} = \sqrt{\frac{1+2\tilde{x} - \sqrt{\tilde{x}(4+5\tilde{x})}}{1+\tilde{x}}} \equiv \sqrt{\frac{1-\tilde{x}}{1+2\tilde{x}+\sqrt{\tilde{x}(4+5\tilde{x})}}} \ , \quad \tilde{x} = \frac{(1-\bar{\mu}^2)^2}{1+4\bar{\mu}^2 - \bar{\mu}^4} \ , \qquad (22)$$

$$k_\infty = \frac{\bar{\mu}(3-\bar{\mu}^2)}{1+\bar{\mu}^2} \ , \quad k_0 = \bar{\mu}(4-\bar{\mu}^2) \ , \qquad (23)$$

For simplicity we restrict ourselves to the case of illumination of the sea surface by direct sunlight. In this case

$$L_0^q(\mu,\varphi) = L_0\, \delta(\varphi)\delta(\mu-\mu_s), \quad \mu_s = \sqrt{1 - \frac{\sin^2 z_\oplus}{n_r^2}} \ , \qquad (24)$$

here $z_\oplus$ is the solar zenith angle, n_r is the refractive index of sea water. The source function $g(\tau)$ has the form

$$g(\tau) = \frac{\tilde{x} L_0}{1+\tilde{x}} e^{-\tau/\mu_s} \ , \qquad (25)$$

and the solution of equations (21) with the boundary conditions

$$E_d^s(z=0)=0,\ \lim_{z\to\infty} E_i^s(z)=0 \tag{26a}$$

for an optically infinitely deep sea will be

$$\left.\begin{aligned} E_d^s(\tau) &= A\ (e^{-k_\infty \tau} - e^{-\tau/\mu_s})\ , \\ E_u^s(\tau) &= R_\infty A e^{-k_\infty \tau} - B e^{-\tau/\mu_s}\ , \end{aligned}\right\} \tag{26}$$

here $R_\infty = \left(\dfrac{1-\bar{\mu}}{1+\bar{\mu}}\right)^2$ is the diffuse reflectance (DR) in the asymptotic regime, and

$$A = \frac{\tilde{x} L_0 (2+\bar{\mu}+1/\mu_s)}{(1+\tilde{x})(1/\mu_s - k_\infty)(1/\mu_s + k_0)}\ ,\ B = \frac{\tilde{x} L_0 (2-\bar{\mu}-1/\mu_s)}{(1+\tilde{x})(1/\mu_s - k_\infty)(1/\mu_s + k_0)} \quad , \text{(26b)}$$

or

$$\left.\begin{aligned} E_d^s(\tau) &= L_0 \mu_s R_s \frac{Q_s m_1}{1-\mu_s k_\infty}\ (e^{-k_\infty \tau} - e^{-\tau/\mu_s})\ , \\ E_u^s(\tau) &= L_0 \mu_s R_s \left[e^{-k_\infty \tau} + \frac{Q_s m_2}{1-\mu_s k_\infty}\ (e^{-k_\infty \tau} - e^{-\tau/\mu_s}) \right], \end{aligned}\right\} \tag{27}$$

here $m_1 = 1+\mu_s(2+\bar{\mu})$, $m_2 = \mu_s(2-\bar{\mu})-1$, $Q_s = \dfrac{1}{1+R_\infty} \equiv \dfrac{(1+\bar{\mu})^2}{2(1+\bar{\mu}^2)}$,

$$R_s = \frac{(1-\bar{\mu})^2}{1+\mu_s k_0} \equiv \frac{(1-\bar{\mu})^2}{1+\mu_s \bar{\mu}(4-\bar{\mu}^2)}\ . \tag{28}$$

here R_s is the diffuse reflectance of the sea illuminated by direct sunlight. *Unscattered* irradiances are

$$E_d^q(\tau) = L_0 \mu_s e^{-\tau/\mu_s}, \qquad E_u^q(\tau) \equiv 0. \tag{29}$$

We can rewrite the total elastic irradiances $E_i^E = E_i^q + E_i^s$ and their derivatives in a form convenient for a numerical computation:

$$E_d^E(z) = L_0\, \mu_s\, e^{-\nu z} \left[e^{2 z_s} + R_s \eta\, Q_s\, m_1 D_0(z) \right]\ , \tag{30}$$

$$E_u^E(z) = L_0\, \mu_s\, e^{-\nu z} R_s \left[1 + \eta\, Q_s\, m_2 D_0(z) \right]\ , \tag{30a}$$

$$-\frac{dE_d^E(z)}{dz} = L_0\, \mu_s\, e^{-\nu z}\, \eta \left[e^{2 z_s} + R_s\, Q_s\, m_1 D_z(z) \right]\ , \tag{31}$$

$$-\frac{dE_u^E(z)}{dz} = L_0\, \mu_s\, e^{-\nu z} R_s \left[\nu + \eta\, Q_s\, m_2 D_z(z) \right]\ , \tag{31a}$$

here $\eta = \alpha/\mu_s$; $\nu = \alpha k_\infty$; $z_s = (\nu - \eta) z/2$; $D_0(z) = (z/2)\left(e^{2 z_s} + 1\right) f_\tau(z_s)$; $f_\tau(x) = \tanh(x)/x$; $D_z(z) = \nu D_0(z) - e^{2 z_s}$.

Downward and upward elastic irradiance attenuation coefficients will be

$$k_d^E(z) \;=\; -\frac{d\ln E_d^E(z)}{dz} \;=\; \eta\,\frac{e^{2z_s} + R_s\,Q_s\,m_1 D_z(z)}{e^{2z_s} + R_s\,\eta Q_s\,m_1 D_0(z)}\;, \tag{32}$$

$$k_u^E(z) \;=\; -\frac{d\ln E_u^E(z)}{dz} \;=\; \frac{\nu + \eta Q_s\,m_2 D_z(z)}{1 + \eta Q_s\,m_2 D_0(z)}\;. \tag{32a}$$

5. Approximate Solutions for Inelastic Scattering (Raman Scattering and Fluorescence)

Equation (6) in full form is

$$\left(\mu\frac{d}{d\tau}+1\right)L^I(\tau,\mu,\varphi) = \frac{1}{2\pi}\left[\frac{\tilde{x}}{1+\tilde{x}}\int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu' L^I(\tau,\mu',\varphi') + \Delta(\tau,\mu,\varphi) + g^{sI}(\tau,\mu,\varphi) + g^{qI}(\tau,\mu,\varphi)\right], \tag{33}$$

or

$$\hat{T}_\Delta L^I \;=\; Q^I, \quad Q^I \;=\; \frac{1}{2\pi}(g^{sI} + g^{qI})\;, \tag{34}$$

here

$$g^{sR}(\lambda,\tau,\mu,\varphi) \;=\; \frac{3}{8\alpha(\lambda)}\int d\lambda'\phi^R(\lambda',\lambda)\int_0^{2\pi} d\varphi'\int_{-1}^{1} d\mu'(1+\cos^2\gamma)L^s(\lambda',\tau,\mu',\varphi')\;, \tag{35}$$

$$g^{sF}(\lambda,\tau,\mu,\varphi) \;=\; \frac{1}{2\alpha(\lambda)}\int d\lambda'\phi^F(\lambda',\lambda)\int_0^{2\pi} d\varphi'\int_{-1}^{1} d\mu'\,L_0^s(\lambda',\mu',\varphi')\;, \tag{36}$$

$$g^{qR}(\lambda,\tau,\mu,\varphi) \;=\; \frac{3}{8\alpha(\lambda)}\int d\lambda'\phi^R(\lambda',\lambda)\int_0^{2\pi} d\varphi'\int_{0}^{1} d\mu'(1+\cos^2\gamma)L_0^q(\lambda',\mu',\varphi')e^{-\tau(\lambda')/\mu'}\;, \tag{37}$$

$$g^{qF}(\lambda,\tau,\mu,\varphi) \;=\; \frac{1}{2\alpha(\lambda)}\int d\lambda'\phi^F(\lambda',\lambda)\int_0^{2\pi} d\varphi'\int_{0}^{1} d\mu'\,L_0^q(\lambda',\mu',\varphi')e^{-\tau(\lambda')/\mu'}\;, \tag{38}$$

$$\left.\begin{aligned} \phi^R(\lambda',\lambda) \;&=\; b^R(\lambda)\; f^R(\lambda',\lambda)\,, \\ \phi^F(\lambda',\lambda) \;&=\; b^F(\lambda') f^F(\lambda',\lambda)\,, \end{aligned}\right\} \tag{39}$$

$$\cos\gamma \;\equiv\; \mathbf{nn'} \;=\; \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi-\varphi')\;. \tag{40}$$

Equations (34) are identical to equation (14) with different right–hand sides. They can be reduced to equations for irradiances in a similar manner. We now have

$$\hat{L}_{ik}(\tau)\;\; E_k^I(\tau) \;=\; e_i g^I(\tau)\;, \quad e_i \;=\; (1,\; 1)\;, \quad i, k = d, u \;\text{ or } 1, 2\;, \tag{41}$$

with the boundary conditions $E_d^I(z=0)=0$, $\lim_{z\to\infty} E_i^I(z)=0$. Here the matrix operator $\hat{L}_{ik}(\tau)$ has the form of (21) and right–hand sides $g^I(\tau)$ of (41) are

$$g^I(\lambda,z) = \frac{1}{2\alpha(\lambda)} \int \phi^I(\lambda',\lambda)\Big\{L_0(\lambda')e^{-\alpha(\lambda')z/\mu_s} + +[2-\bar{\mu}(\lambda')]E_d^s(\lambda',z) + [2+\bar{\mu}(\lambda')]E_u^s(\lambda',z)\Big\}d\lambda', \quad (42)$$

irradiances E_d^s and E_u^s are given by Eqns. (27).

The solution of (41) can be represented as

$$E_i^I(\tau) = A^I a_i e^{-k_\infty \tau} + \int_0^\infty \tilde{G}_i(\tau-\tau')g^I(\tau')d\tau' , \quad (43)$$

with $a_i = (1,\ R_\infty)$, $\tilde{G}_i(\tau) = G_{ik}(\tau)e_k$, constant A^I is determined from the boundary conditions, and $G_{ik}(\tau)$ is the Green's function matrix [9] which satisfies the equation

$$\hat{L}_{ik}(\tau)\ G_{kl}(\tau) = \delta_{il}\,\delta(\tau) . \quad (44)$$

where δ_{ik} is the Kronecker symbol. It is not difficult to show, that

$$G_{ik}(\tau) = \begin{Vmatrix} 1 & R_0 \\ R_\infty & R_0R_\infty \end{Vmatrix} \frac{\theta(\tau)e^{-k_\infty\tau}}{1-R_0R_\infty} + \begin{Vmatrix} R_0R_\infty & R_0 \\ R_\infty & 1 \end{Vmatrix} \frac{\theta(-\tau)e^{k_0\tau}}{1-R_0R_\infty} , \quad (45)$$

and

$$\tilde{G}_i(\tau) = k_1[a_i\,k_2\,\theta(\tau)e^{-k_\infty\tau} + b_i\,\theta(-\tau)e^{k_0\tau}] , \quad (46)$$

here $R_0 = \dfrac{2+\bar{\mu}}{2-\bar{\mu}}R_\infty$, $b_i = (R_0,\ 1)$, $k_1 = \dfrac{1+R_\infty}{1-R_0R_\infty}$, $k_2 = \dfrac{1+R_0}{1+R_\infty}$, where $\theta(x)$ is a Heavyside function. Solutions to Eqn. (41) with boundary condition $E_d^I(0)=0$, and vanishing at infinite depths are

$$E_i^I(\tau) = k_1\left\{a_i\left[k_2\int_0^\tau g^I(\tau')e^{-k_\infty(\tau-\tau')}d\tau' - R_0e^{-k_\infty\tau}\int_0^\infty g^I(\tau')e^{-k_0\tau'}d\tau'\right] + b_i\int_\tau^\infty g^I(\tau')e^{k_0(\tau-\tau')}d\tau'\right\}. \quad (47)$$

Eqn. (47) together with Eqns (30) and (30a) for elastic irradiances give the solution to our problem. Unfortunately (47) contains singularities which make it unsuitable for numerical calculations without further analytical modification. This task can be easily performed and the results are

$$E_i^I(\lambda,z) = \frac{k_1}{2}e^{-\nu z}\big\langle a_i(k_2D_R - R_0L_R) + b_iL_N\big\rangle^I , \quad (48)$$

$$-\frac{dE_i^I(\lambda,z)}{dz} = \frac{k_1}{2}e^{-\nu z}\big\langle \nu[a_i(k_2D_R - R_0L_R) + b_iL_N] + +(b_i - k_2a_i)[(1+kD_3)e^{2r_1} + ne^{2r_2}] - (\nu+\zeta)b_iL_N\big\rangle^I , \quad (49)$$

here $l_1 = 1/(\zeta+\eta')$, $\zeta = \alpha k_0$, $l_2 = 1/(\zeta+\nu')$, $r_1 = (\nu-\eta')z/2$, $r_2 = (\nu-\nu')z/2$, $r_3 = r_2 - r_1$, $D_R = D_1 + nD_2 + kS$, $n = \mu_s R_s'(2+\bar{\mu}')$, $S = (z^2/4)\big[(e^{2r_1}+e^{2r_2})f_\tau(r_1)f_\tau(r_3) - (e^{2r_2}+1)f_{xy}(r_1,r_2)\big]$, $D_i = (z/2)(e^{2r_i}+1)f_\tau(r_i)$, $i=1,2,3$, $k = 2\mu_s R_s' Q_s' \eta'[\mu_s(4-\bar{\mu}'^2) - \bar{\mu}']$, $L_1 = l_1e^{2r_1}$, $L_K = kD_3L_1$, $L_R = l_1 + l_2(n+kl_1)$, $L_N = L_1 + l_2N_K + L_K$, $N_K = (n+kl_1)e^{2r_2}$,

$f_{xy}(x,y) = \frac{1}{y-x}\left[\frac{\tanh(x)}{x} - \frac{\tanh(y)}{y}\right]$, $\langle \Psi(\lambda',\lambda)\rangle^I \equiv \int d\lambda' \phi^I(\lambda',\lambda) L_0(\lambda')\Psi(\lambda',\lambda)$, here $\Psi(\lambda',\lambda)$ is any function.

The total irradiance attenuation coefficient (IAC) will be calculated according to [10, 11]

$$k_i \;=\; -\frac{d}{dz}\ln\left[E_i^E(z) + E_i^R(z) + \sum_F E_i^F(z)\right], \tag{50}$$

or

$$k_d = \nu + \frac{\left\langle (R_0 - k_2)\left[(1+kD_3)e^{2r_1} + ne^{2r_2}\right] - (\nu+\zeta)R_0 L_N \right\rangle^I}{\left\langle k_2 D_R - R_0 L_R + R_0 L_N \right\rangle^I}, \tag{50a}$$

$$k_u = \nu + \frac{\left\langle (1 - k_2 R_\infty)\left[(1+kD_3)e^{2r_1} + ne^{2r_2}\right] - (\nu+\zeta)L_N \right\rangle^I}{\left\langle R_\infty(k_2 D_R - R_0 L_R) + L_N \right\rangle^I}. \tag{50b}$$

The total diffuse reflectance will be

$$R \equiv \frac{E_u(z=0)}{E_d(z=0)} = R_s + \sum_I \delta R^I, \tag{51}$$

where

$$\delta R^I = \frac{1+R_\infty}{2L_0\mu_s}\left\langle l_1 + l_2(n + kl_1)\right\rangle^I. \tag{51a}$$

6. Model Calculations

6.1. Model of Sea Water Optical Properties.

We adopt the following model [2, 12]:
the absorption coefficient (m^{-1}):

$$a(\lambda) = a_w(\lambda) + a_C^0(\lambda)C + a_Y^0(\lambda)Y \ , \tag{52}$$

here $a_w(\lambda)$ is the pure water absorption coefficient (m^{-1}), $a_C^0(\lambda)$ is the specific absorption coefficient of chlorophyll (m^2/mg) (see Figure 1), $a_Y^0(\lambda) = 0.1\exp[0.015(400-\lambda)]$ is the specific absorption coefficient of *yellow substance*, C is the total concentration of chlorophyll (mg/m^3), Y is the concentration of *yellow substance* in relative (dimensionless) units.

Scattering and backscattering coefficients are:

$$b(\lambda) = b_w(\lambda) + b_{ps}^0(\lambda)P_s + b_{pl}^0(\lambda)P_l, \tag{53}$$

$$b_B(\lambda) = \frac{1}{2}b_w(\lambda) + B_s b_{ps}^0(\lambda)P_s + B_l b_{pl}^0(\lambda)P_l, \tag{54}$$

here $b_w(\lambda)$ is scattering coefficient of pure water (m^{-1}) (our interpolation of data given in Ref. [13]):

$$b_w(\lambda) = 5.826\cdot 10^{-3}\left(\frac{400}{\lambda}\right)^{4.322}, \tag{55}$$

$b_{ps}^0(\lambda)$ and $b_{pl}^0(\lambda)$ are scattering coefficients of small and large particulate matter [12]

$$b_{ps}^0(\lambda) = 1.1513\left(\frac{400}{\lambda}\right)^{1.7}, \ (m^2/g), \tag{56}$$

$$b_{pl}^0(\lambda) = 0.3411\left(\frac{400}{\lambda}\right)^{0.3}, \ (m^2/g), \tag{57}$$

$B_s = 0.039$ is backscattering probability of small particles, $B_l = 6.4\cdot 10^{-4}$ is the backscattering probability of large particles, P_s is the concentration of small particles (g/m^3) and P_l is the concentration of large particles (g/m^3). In all our model calculations the wavelength of light is expressed in *nm* and the wavenumber in cm^{-1}.

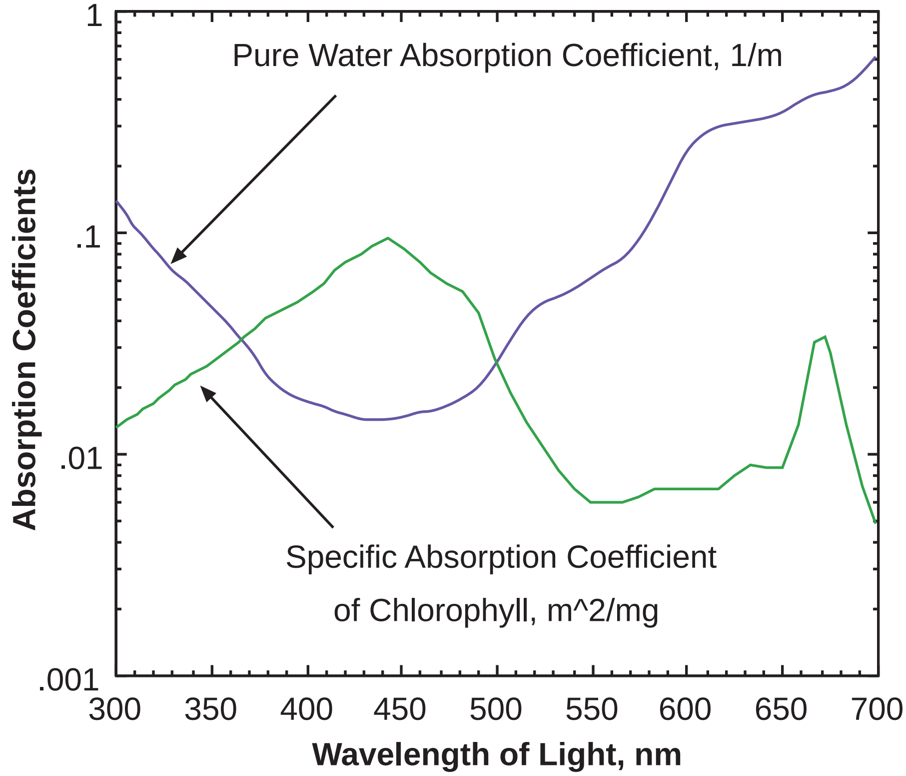

Figure 1. Absorption Coefficient of Pure Water $a_w(\lambda)$ according to Smith and Baker [14] and Specific Absorption Coefficient of Chlorophyll $a_C^0(\lambda)$ according to Yentsch [15].

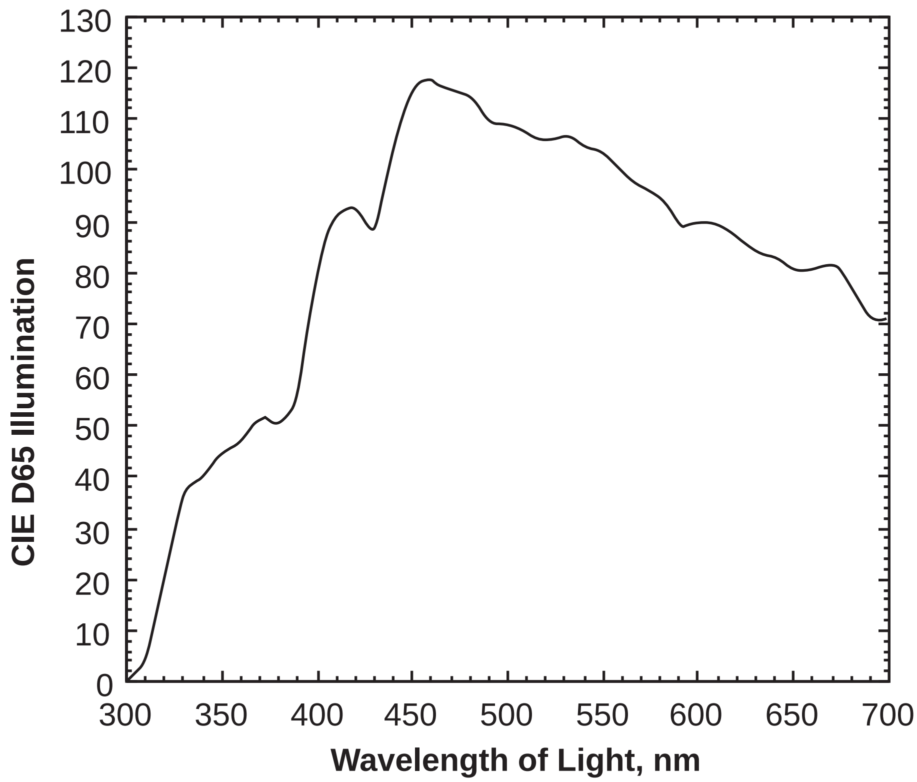

Figure 2. Spectral Irradiance of Sea Level $L_0(\lambda)$ according to Judd and Wyszecki [16] (here $4\pi L_0(\lambda)$ is in $mW\ m^{-2}\ nm^{-1}$).

Table 1. Absorption Coefficient of Pure Water $a_w(\lambda)$ (m^{-1}) according to Smith and Baker [14], Specific Absorption Coefficient of Chlorophyll $a_C^0(\lambda)$ (mg/m^3) according to Yentsch [15] and Spectral Irradiance on Sea Level $L_0(\lambda)$ according to Judd and Wyszecki [16]; ($4\pi L_0(\lambda)$ is in $mW\ m^{-2}\ nm^{-1}$).

l, *nm*	$a_w(\lambda)$	$a_C^0(\lambda)$	$L_0(\lambda)$	l, *nm*	$a_w(\lambda)$	$a_C^0(\lambda)$	$L_0(\lambda)$
290.	.215	.011	0.	500.	.0257	.025	109.4
300.	.141	.013	0.03	510.	.0357	.017	107.8
310.	.105	.015	3.30	520.	.0477	.012	104.8
320.	.0844	.017	20.2	530.	.0507	.009	107.7
330.	.0678	.020	37.1	540.	.0558	.007	104.4
340.	.0561	.023	39.9	550.	.0638	.006	104.0
350.	.0463	.026	44.9	560.	.0708	.006	100.0
360.	.0379	.030	46.6	570.	.0799	.006	96.3
370.	.0300	.035	52.1	580.	.108	.007	95.8
380.	.0220	.042	50.0	590.	.157	.007	88.7
390.	.0191	.046	54.6	600.	.244	.007	90.0
400.	.0171	.051	82.8	610.	.289	.007	89.6
410.	.0162	.060	91.5	620.	.309	.007	87.7
420.	.0153	.074	93.4	630.	.319	.009	83.3
430.	.0144	.080	86.7	640.	.329	.009	83.7
440.	.0145	.097	104.9	650.	.349	.008	80.0
450.	.0145	.087	117.0	660.	.400	.014	80.2
460.	.0156	.074	117.8	670.	.430	.041	82.3
470.	.0156	.060	114.9	680.	.450	.017	78.3
480.	.0176	.058	115.9	690.	.500	.008	69.7
490.	.0196	.044	108.8	700.	.650	.004	71.6

6.2. Raman Scattering Model.

The frequency redistribution of Raman scattered light was represented according to the Walrafen data [17], namely

$$f^R(\nu'_f,\nu_f) = k^R \sum_{i=1}^{4} \alpha_i \exp\left[-\frac{(\nu'_f-\nu_f-\Delta\nu_i)^2}{2\sigma_i^2}\right], \tag{58}$$

here ν'_f is the excitation wavenumber, ν_f is the emission wavenumber ($\nu_f = 10^7/(2\pi\lambda)$, ν_f is in cm^{-1}, λ is in nm), $k^R = \left(\sqrt{2\pi}\sum_{i=1}^{4}\alpha_i\sigma_i\right)^{-1} = 5.152\cdot10^{-3}$. Values of α_i, $\Delta\nu_i$ and σ_i are given in the following table:

Table 2

i	α_i	$\Delta\nu_i$	σ_i
1	0.41	562.1	89.18
2	0.39	387.1	74.32
3	0.10	282.1	59.45
4	0.10	187.1	59.45

We represent the inelastic Raman scattering coefficient as [17, 18]

$$b^R(\nu'_f,\nu_f,\cos\gamma) \; = \; \beta_0^R \left(\frac{\nu_f}{\nu_f^0}\right)^4 f^R(\nu'_f,\nu_f)\, p_R(\cos\gamma), \qquad (59)$$

here $\beta_0^R \; = \; 0.0004$, $\nu_f^0 \; = \; 3978.87\, cm^{-1}$ (corresponds to 400 *nm*), $p_R(\cos\gamma)$ is the Rayleigh phase function of scattering (we adopted here a Rayleigh–type angular scattering distribution).

6.3. Fluorescence Model.

Frequency distribution of light emitted due to fluorescence of chlorophyll *a* we represent according to data given in Refs [19, 20]:

$$f^C(\nu'_f,\nu_f) \; \equiv \; f^C(\nu_f) = k^C \exp\left[-\frac{(\nu_f - \nu_f^{C0})^2}{2\sigma_C^2}\right], \qquad (60a)$$

with ν_f^{C0} = 2320 cm^{-1}; σ_C = 35.85 cm^{-1}; k^C = 0.01113 *cm*.

We adopt here the model which accepts the following distribution of light emitted due to fluorescence by dissolved organic matter or *yellow substance* [21]:

$$f^Y(\nu'_f,\nu_f) \; \equiv \; f^Y(\nu_f) = k^Y \exp\left[-\frac{(\nu_f - \nu_f^{Y0})^2}{2\sigma_Y^2}\right], \qquad (60b)$$

with ν_f^{Y0} = 3740 cm^{-1} ; σ_Y = 142.38 cm^{-1}; k^Y = 0.00280 *cm*.

The total inelastic fluorescence emittance is represented by

$$b^F(\nu'_f,\nu_f) \; = \; \beta_C^0(\nu'_f)\, a_C^0(\nu'_f)\, C\, f^C(\nu_f) + \beta_Y^0(\nu'_f)\, a_Y^0(\nu'_f)\, Y\, f^Y(\nu_f) \; , \qquad (61)$$

with β_C^0 = 0.0008 between 370 and 685 *nm* and zero elsewhere, and β_Y^0 = 0.0008 between 300 and 400 *nm* and zero elsewhere [19, 21]. The angular distribution of light emitted due to fluorescence is isotropic.

7. Illustrations

In this section we will present results of model calculations for different optical water types. Several different classifications of optical water types exist now [10, 11, 22]. For purposes of convenience in our calculation we have chosen as a reference classification by Man'kovsky [23] which includes chlorophyll concentration as its parameter. The distribution of optical water types over the Central Atlantic according to that classification is shown on Figure 3.

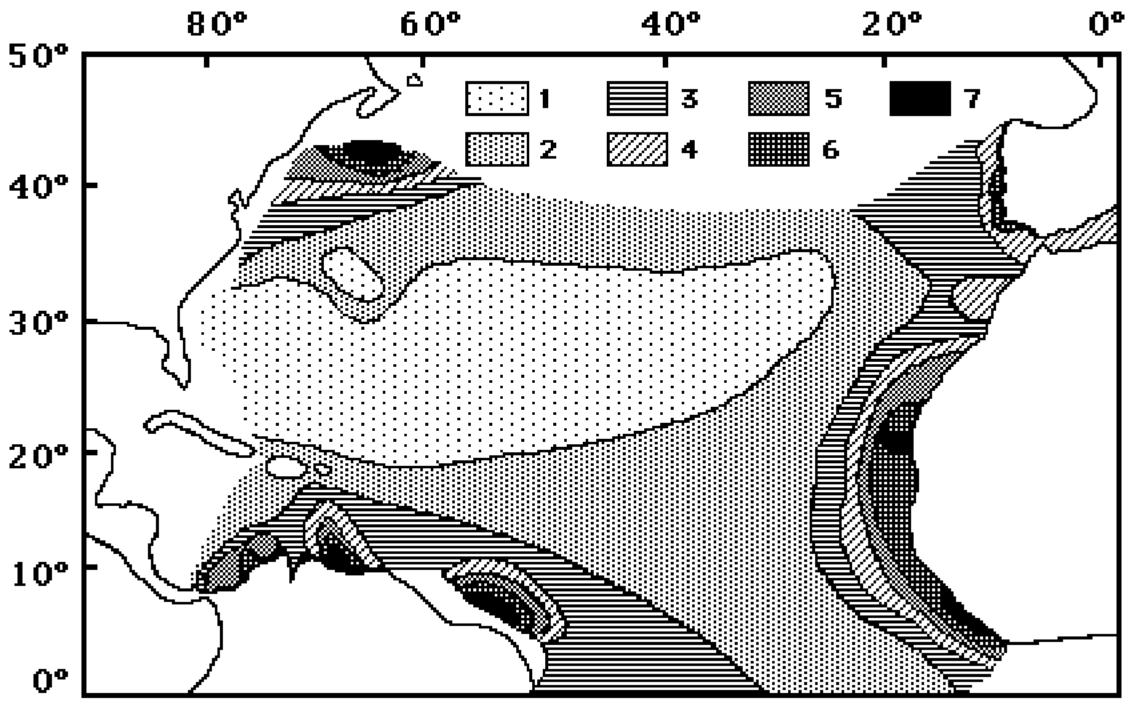

Figure 3. Optical Water Types in Central Atlantic [24] according to classification by Man'kovsky [23]. Numbers correspond to the following concentrations of chlorophyll in *mg/m³*: 1 (<0.18); 2 (0.18—0.30); 3 (0.30—0.46); 4 (0.46—0.67); 5 (0.67—0.87); 6 (0.87—1.20); 7 (>1.20).

Further we will use eight sets of concentrations given in Table 3. The concentrations of *yellow substance* and fractions of particulate matter are chosen according to experimentally derived regressions given in Refs [2, 25].

Table 3.

C, mg/m^3	Y	P, g/m^3	P_L, g/m^3	P_S, g/m^3
0.00	0.00	0.000	0.000	0.000
0.03	0.03	0.036	0.035	0.001
0.05	0.05	0.053	0.051	0.002
0.12	0.12	0.102	0.098	0.004
0.30	0.30	0.203	0.194	0.009
0.60	0.60	0.341	0.325	0.016
1.00	1.00	0.500	0.476	0.024
3.00	3.00	1.140	1.078	0.062

Figures 4–8 show elastic optical properties of waters characterized by Table 3.

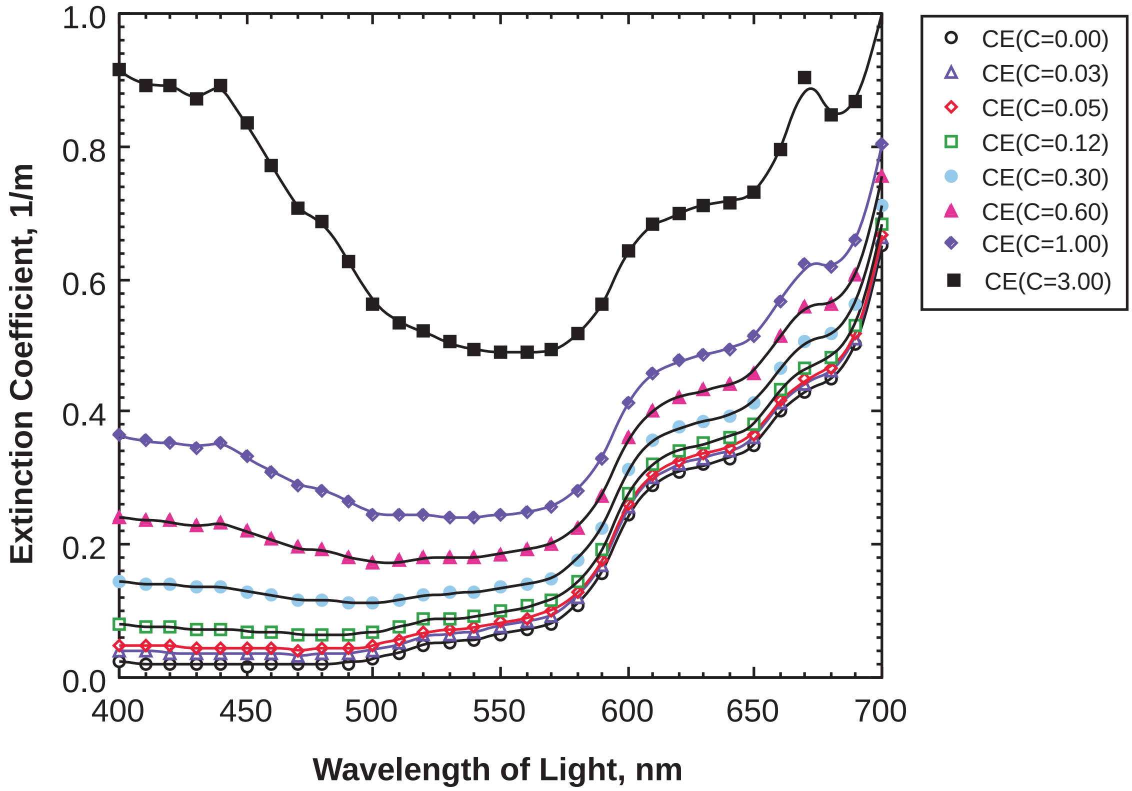

Figure 4. Spectral dependence of Extinction Coefficient c^E for sample waters.

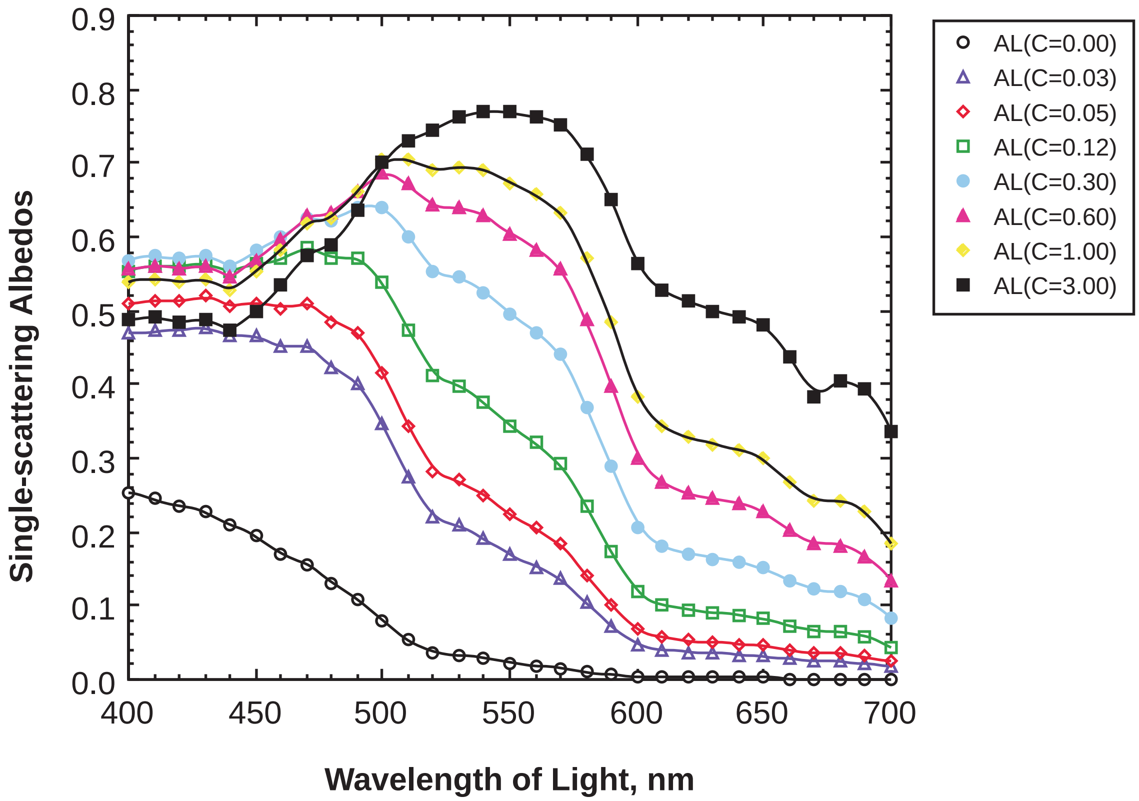

Figure 5. Spectral dependence of Single-scattering Albedos ω_0 for sample waters.

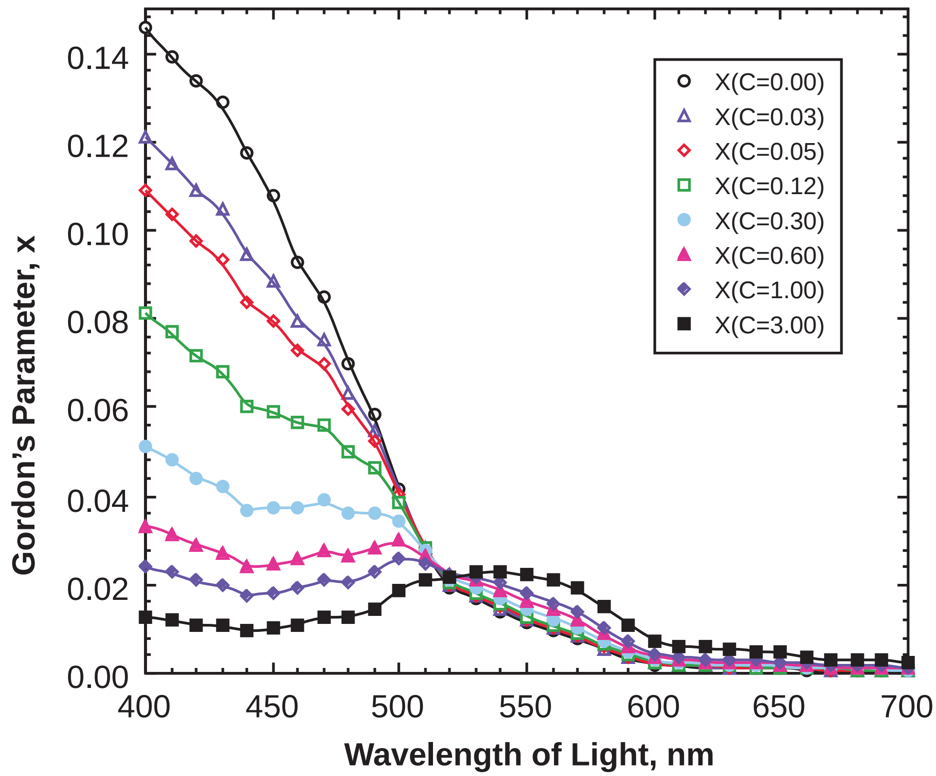

Figure 6. Spectral dependence of Gordon's parameter $x = b_B/(a + b_B)$ for sample waters.

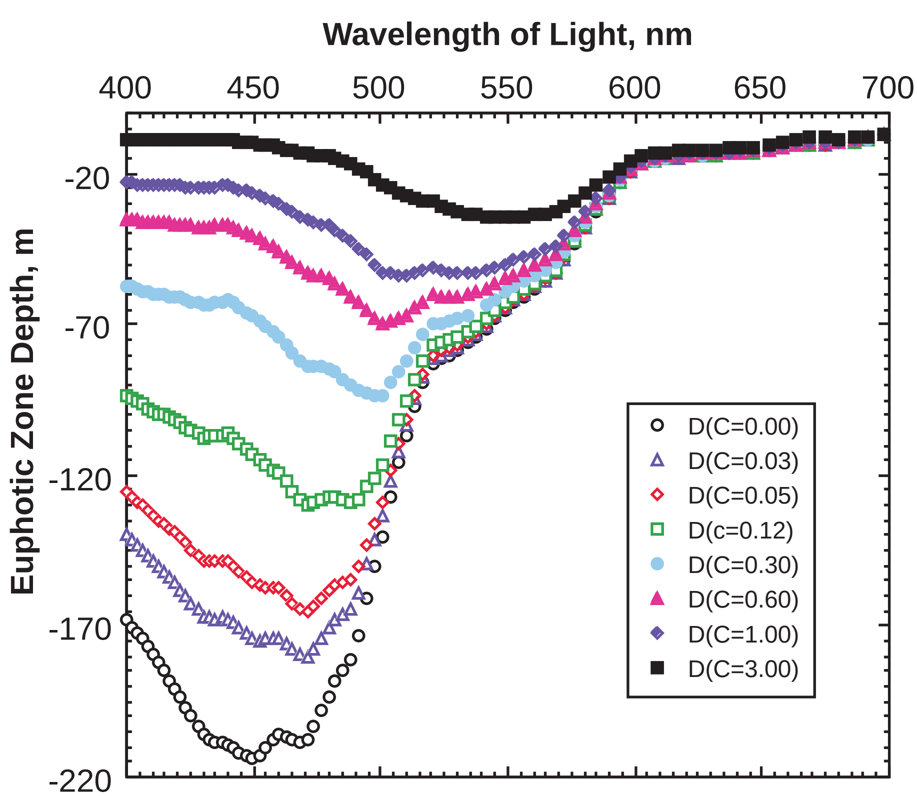

Figure 7. Spectral dependence of Euphotic Zone Depths d (depth of 1% illumination) for sample waters.

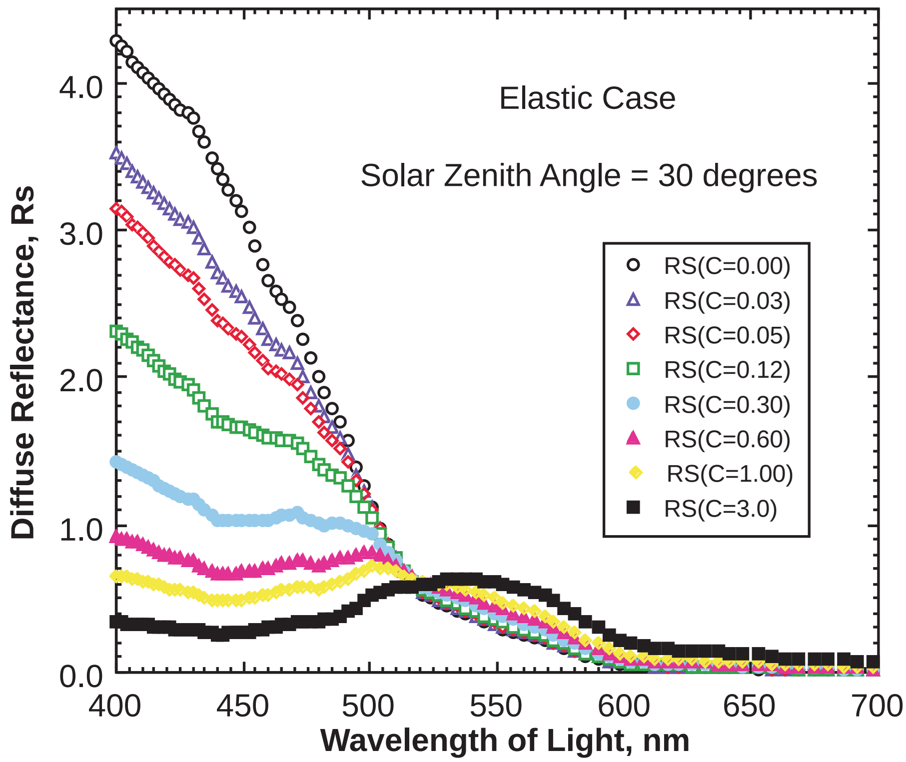

Figure 8. Spectral dependence of Diffuse Reflectance R_S for sample waters.

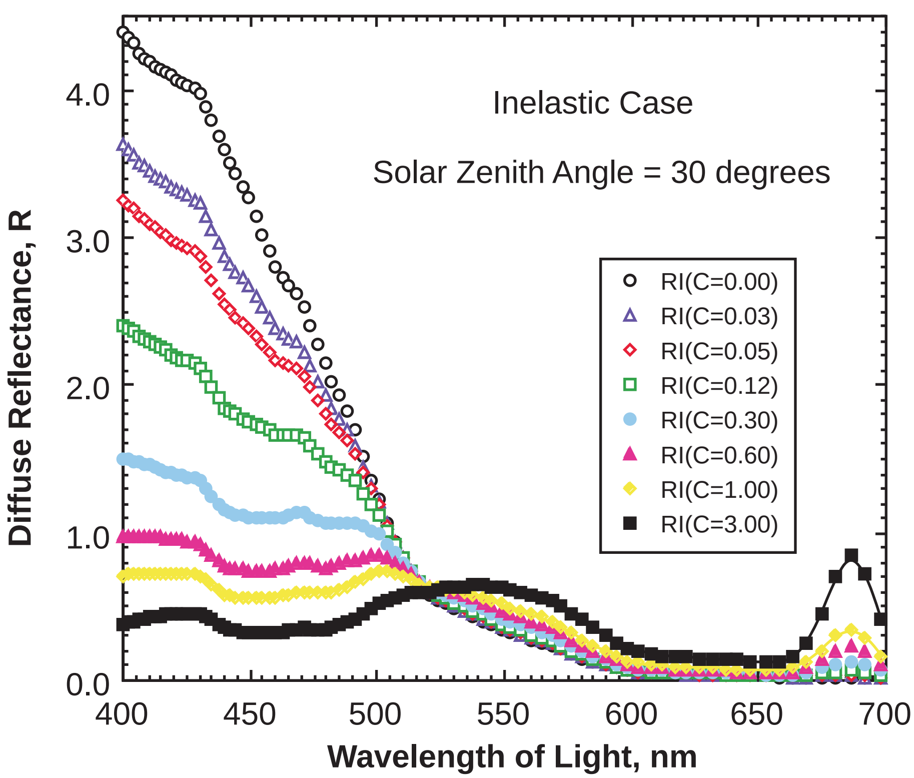

Figure 9. Spectral dependence of Diffuse Reflectance R for sample waters.

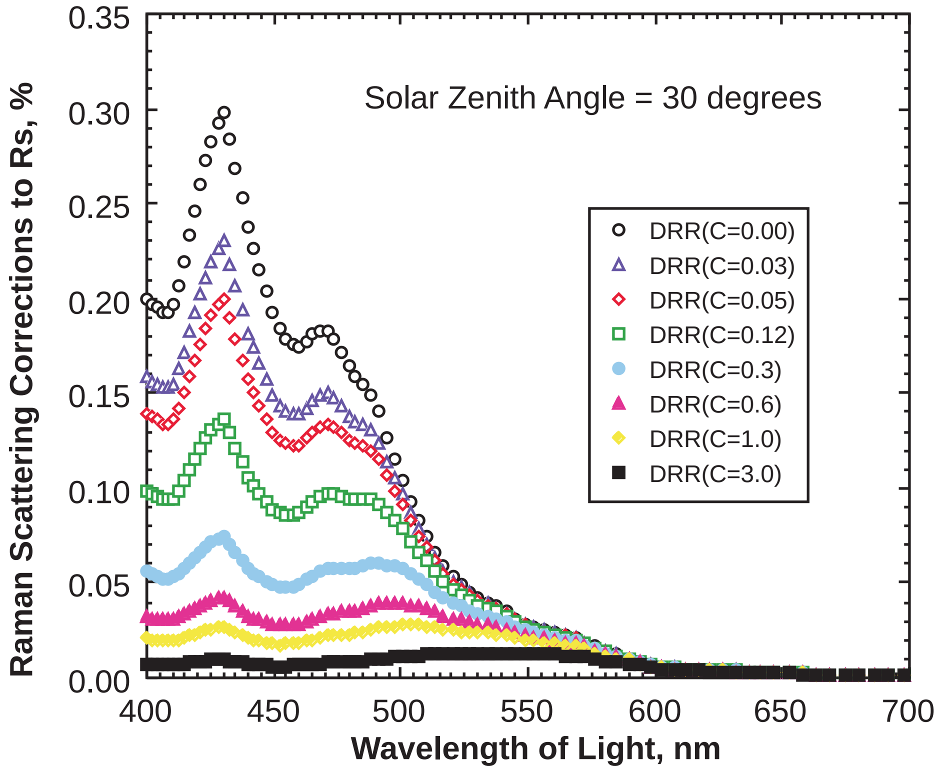

Figure 10. Raman Scattering corrections δR^R to Diffuse Reflectance R_S for sample waters.

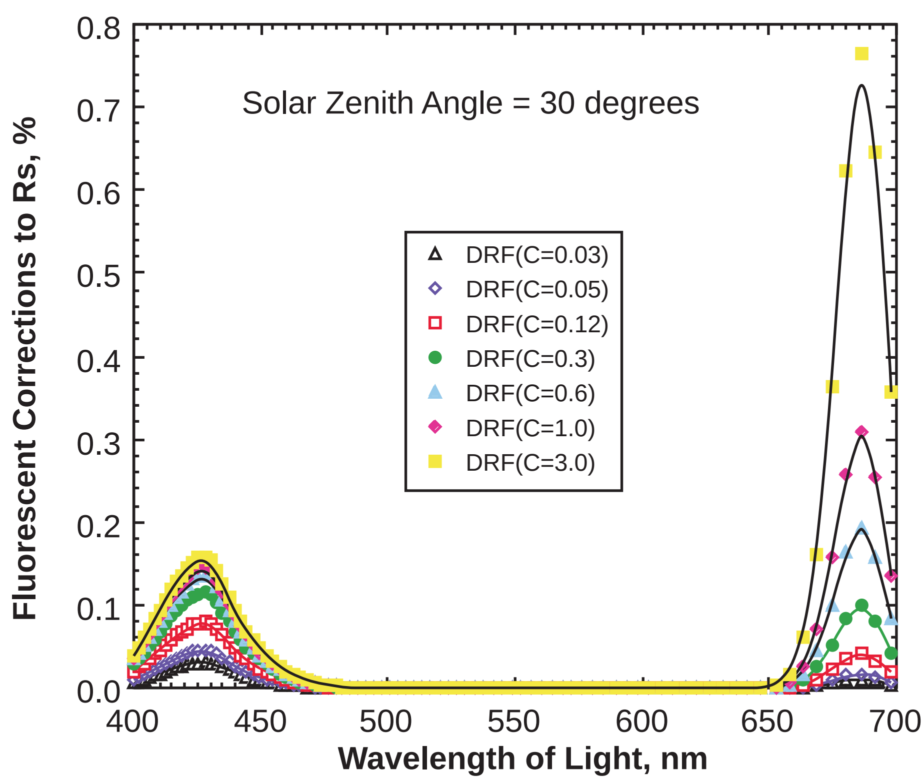

Figure 11. Fluorescent corrections $\delta R^{CY} = \delta R^Y + \delta R^C$ to Diffuse Reflectance R_S for sample waters.

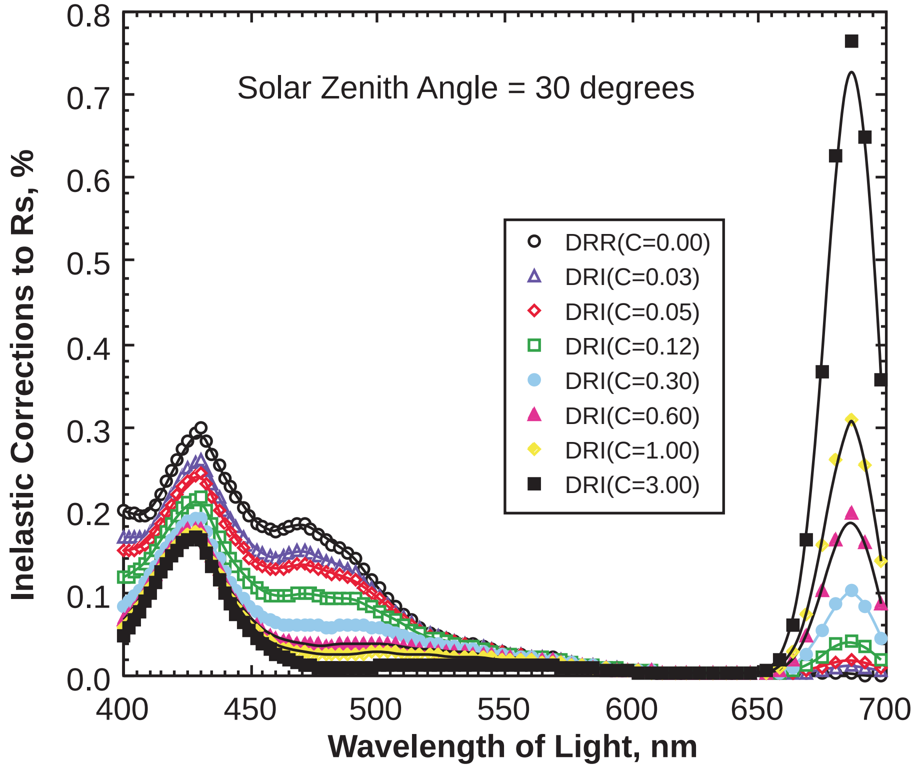

Figure 12. Inelastic corrections $\delta R^R + \delta R^Y + \delta R^C$ (Raman scattering plus *yellow substance* and chlorophyll fluorescence) to Diffuse Reflectance R_S for sample waters.

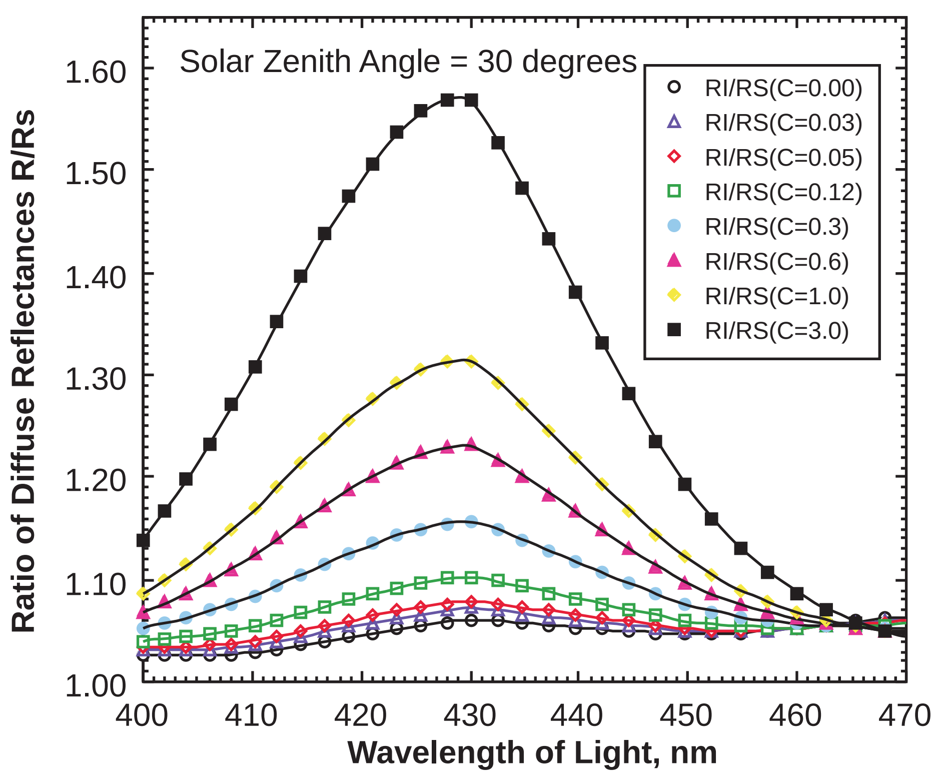

Figure 13. Ratio of inelastic Diffuse Reflectance R to elastic Diffuse Reflectance R_S in blue part of the spectrum for sample waters.

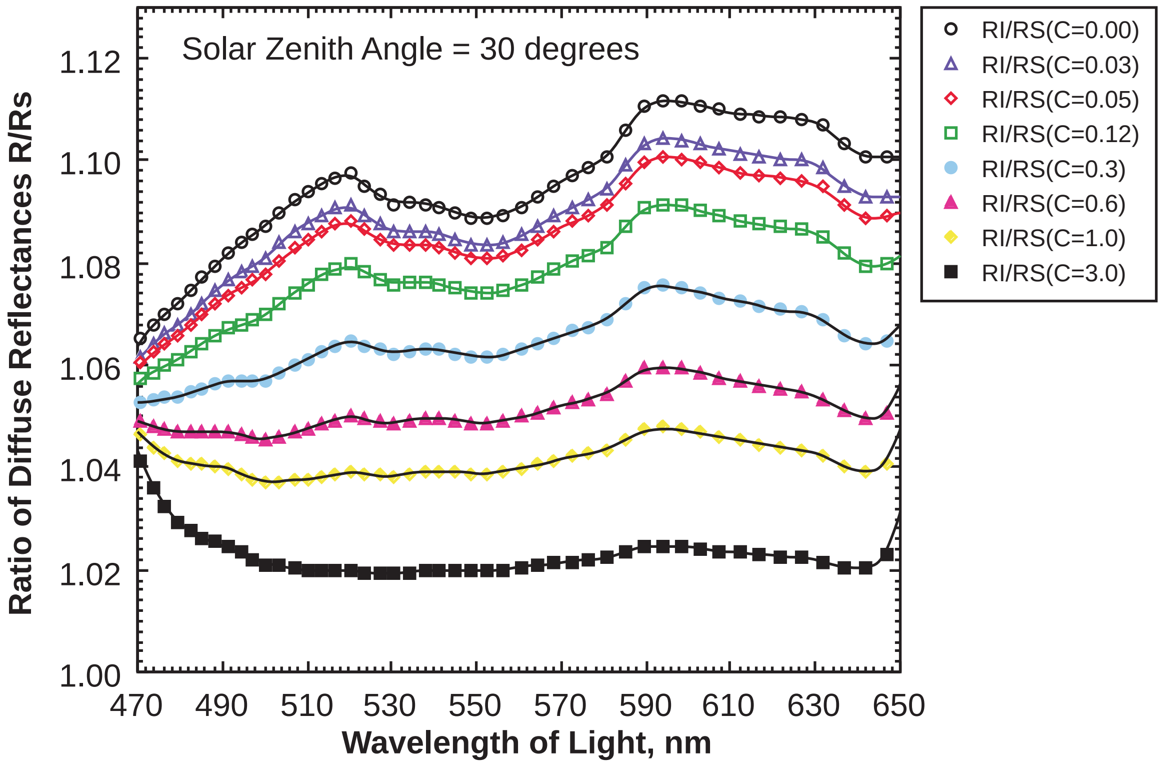

Figure 14. Ratio of inelastic Diffuse Reflectance R to elastic Diffuse Reflectance R_S in green part of the spectrum for sample waters.

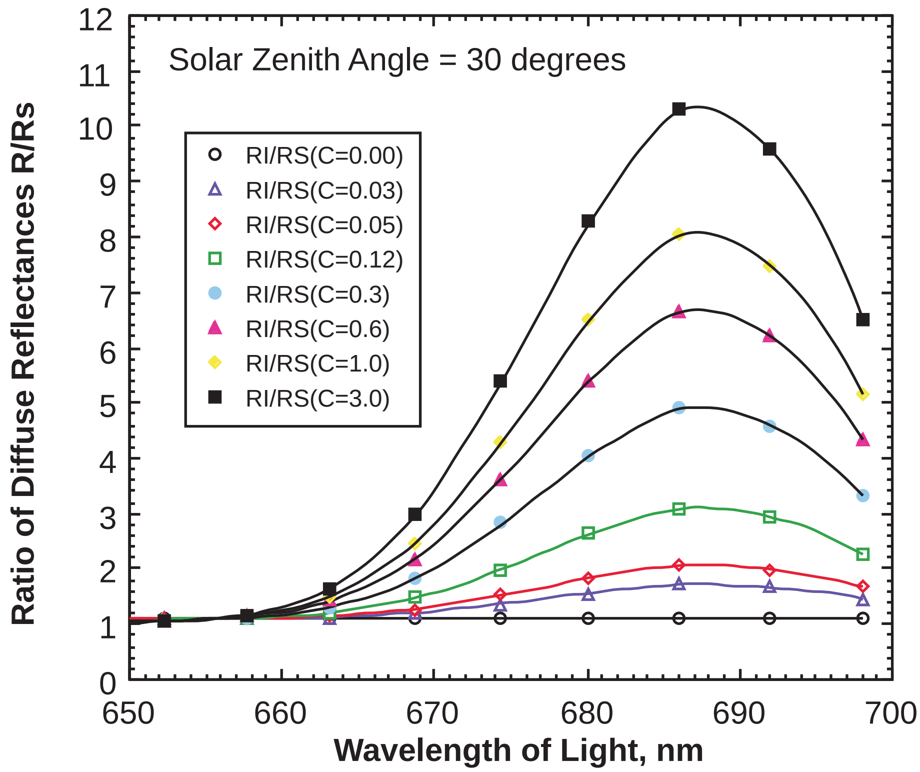

Figure 15. Ratio of inelastic Diffuse Reflectance R to elastic Diffuse Reflectance R_S in red part of the spectrum for sample waters.

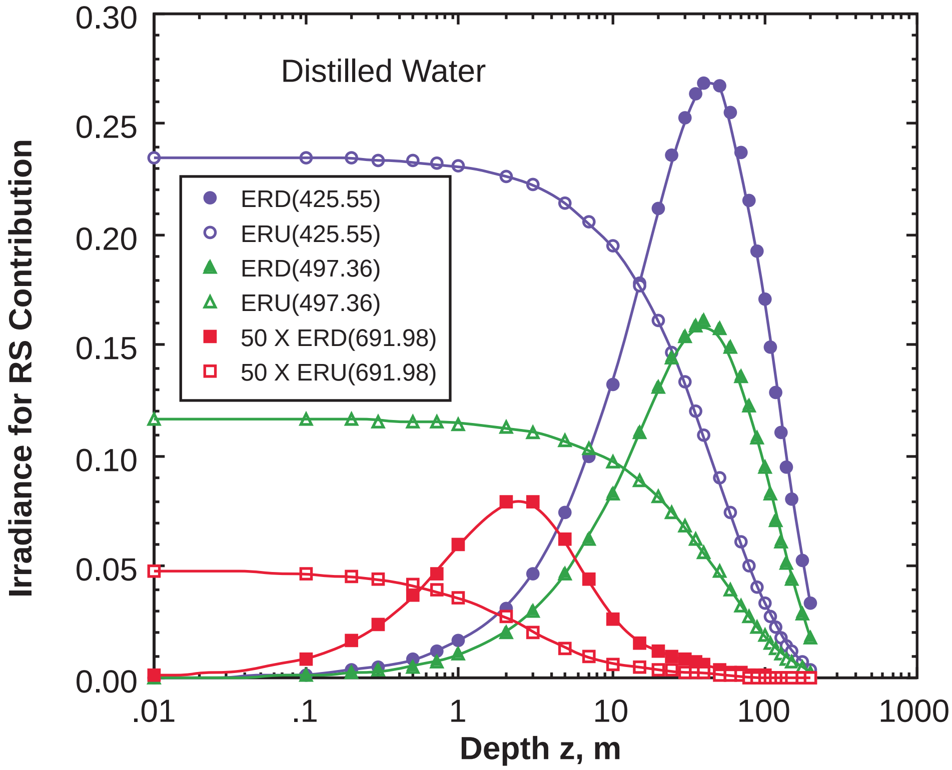

Figure 16. Depth dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in distilled water at different wavelengths.

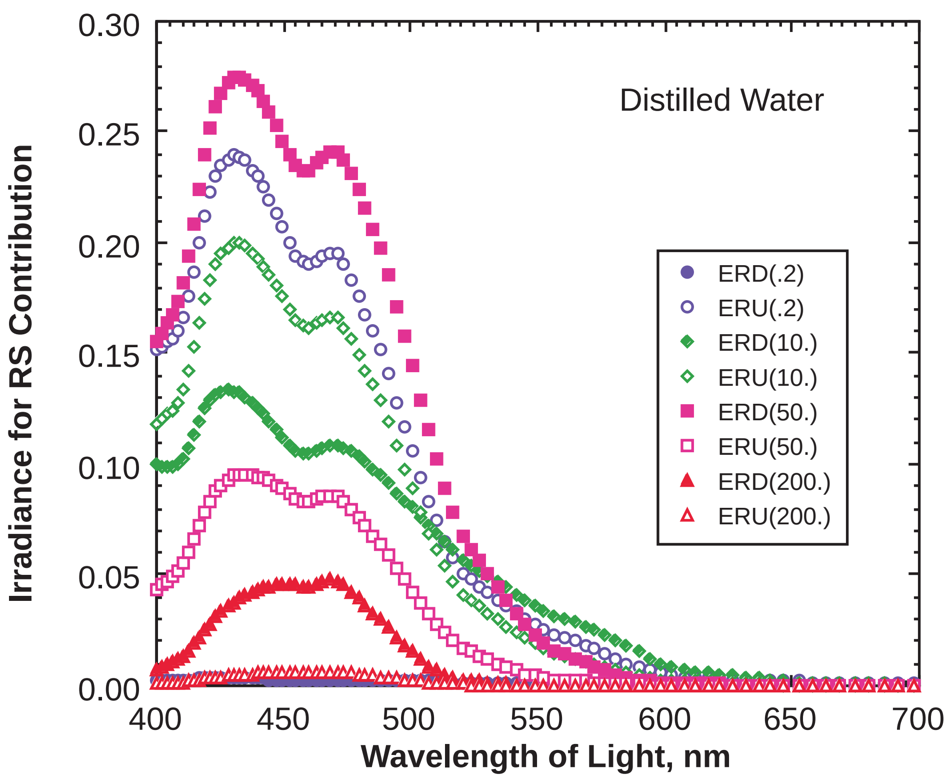

Figure 17. Spectral dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in distilled water at different depths.

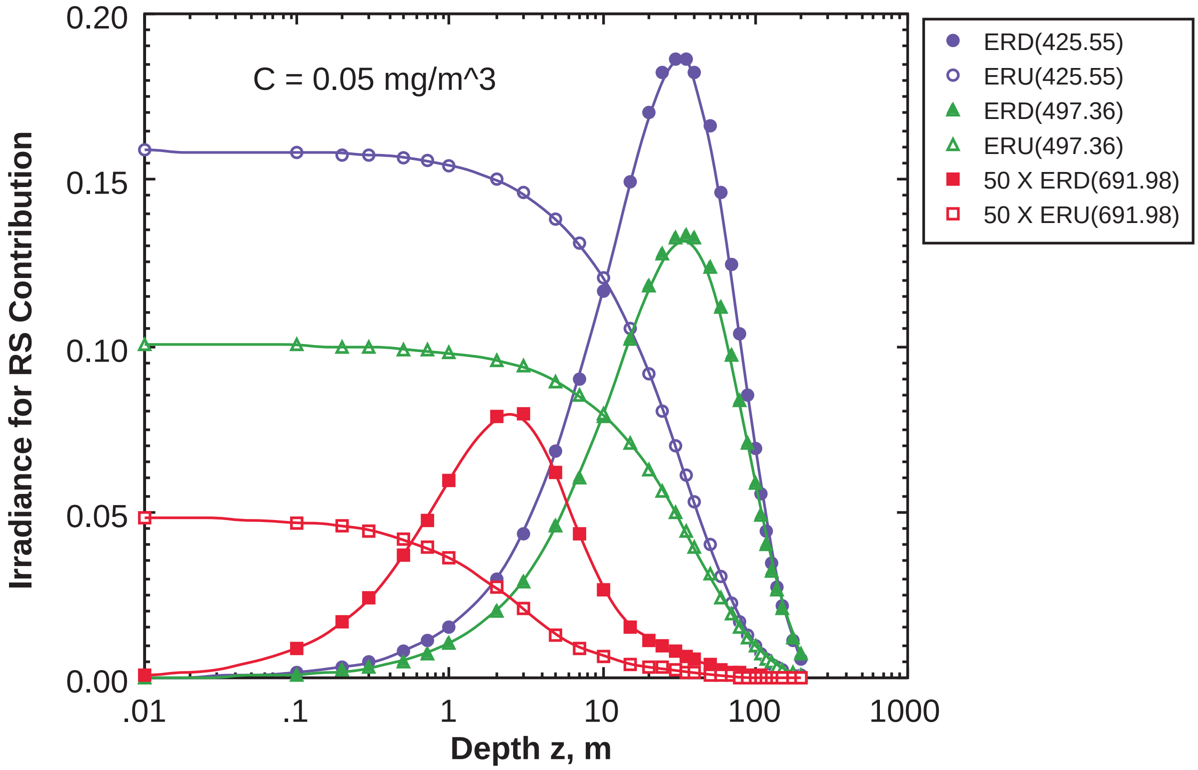

Figure 18. Depth dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in low–productivity water (C=0.05 mg/m^3) at different wavelengths.

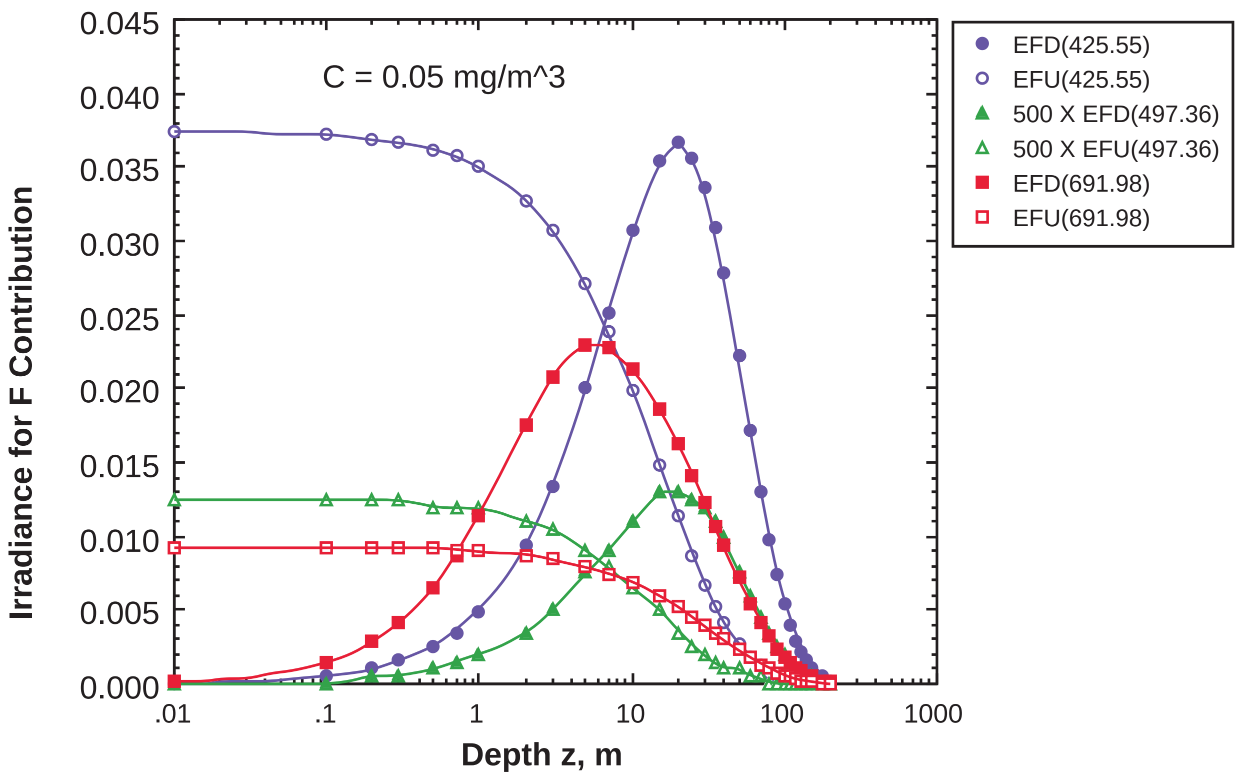

Figure 19. Depth dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in low–productivity water (C=0.05 mg/m^3) at different wavelengths.

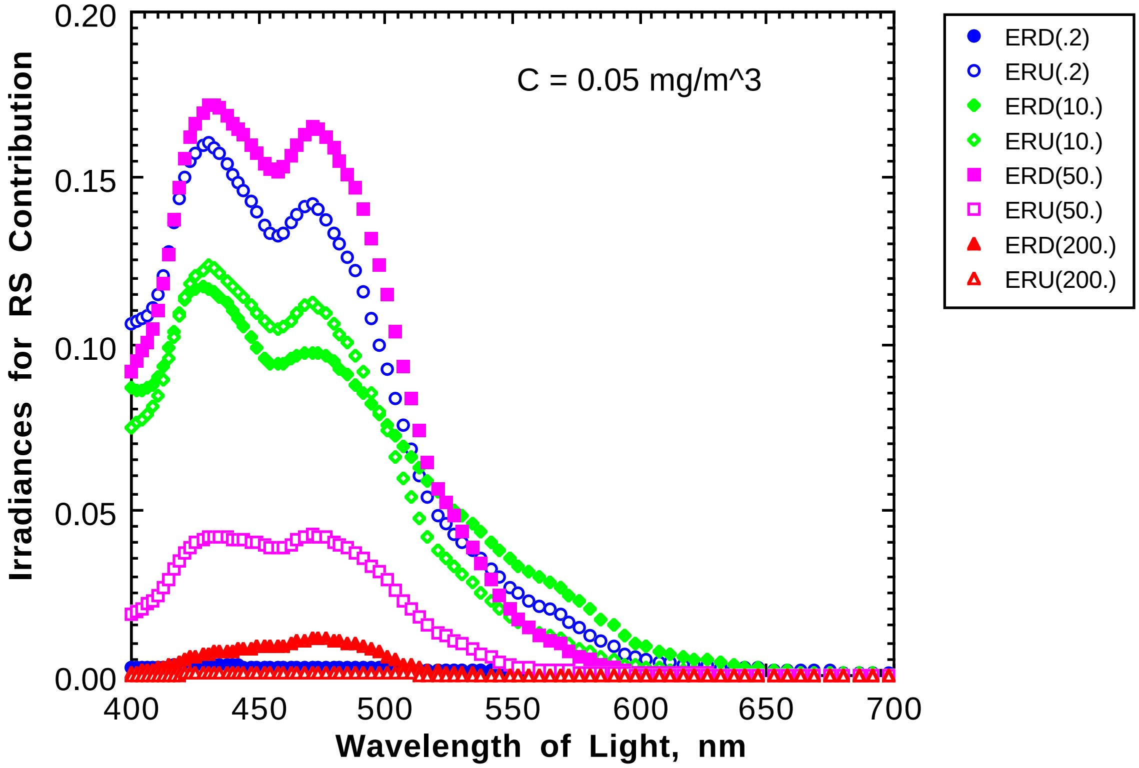

Figure 20. Spectral dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in low–productivity water (C=0.05 mg/m^3) at different depths.

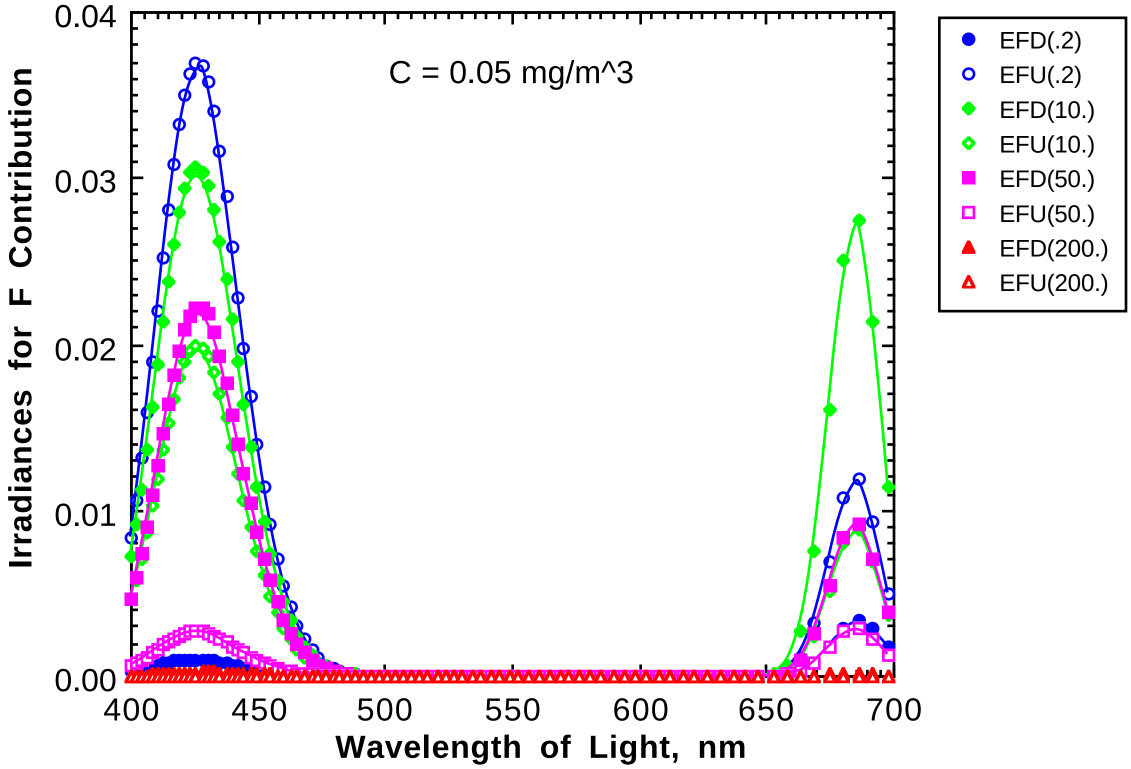

Figure 21. Spectral dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in low–productivity water (C=0.05 mg/m^3) at different depths.

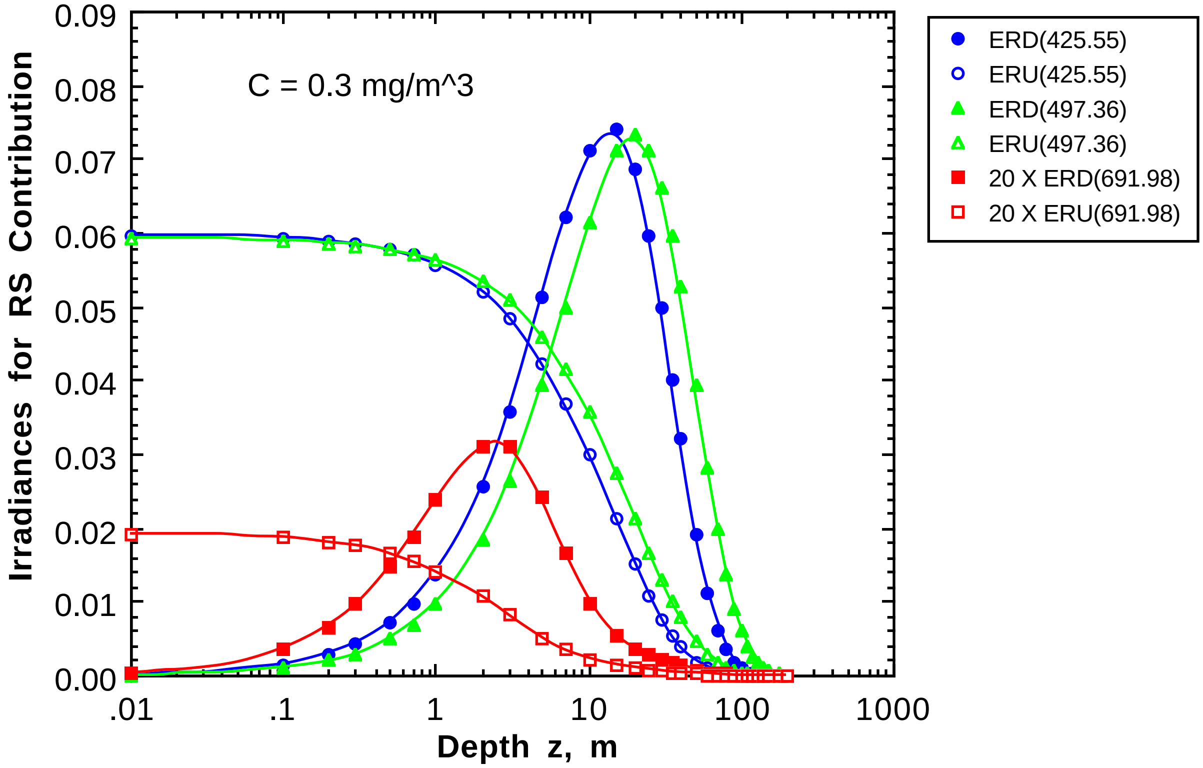

Figure 22. Depth dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in medium–productivity water (C=0.3 mg/m^3) at different wavelengths.

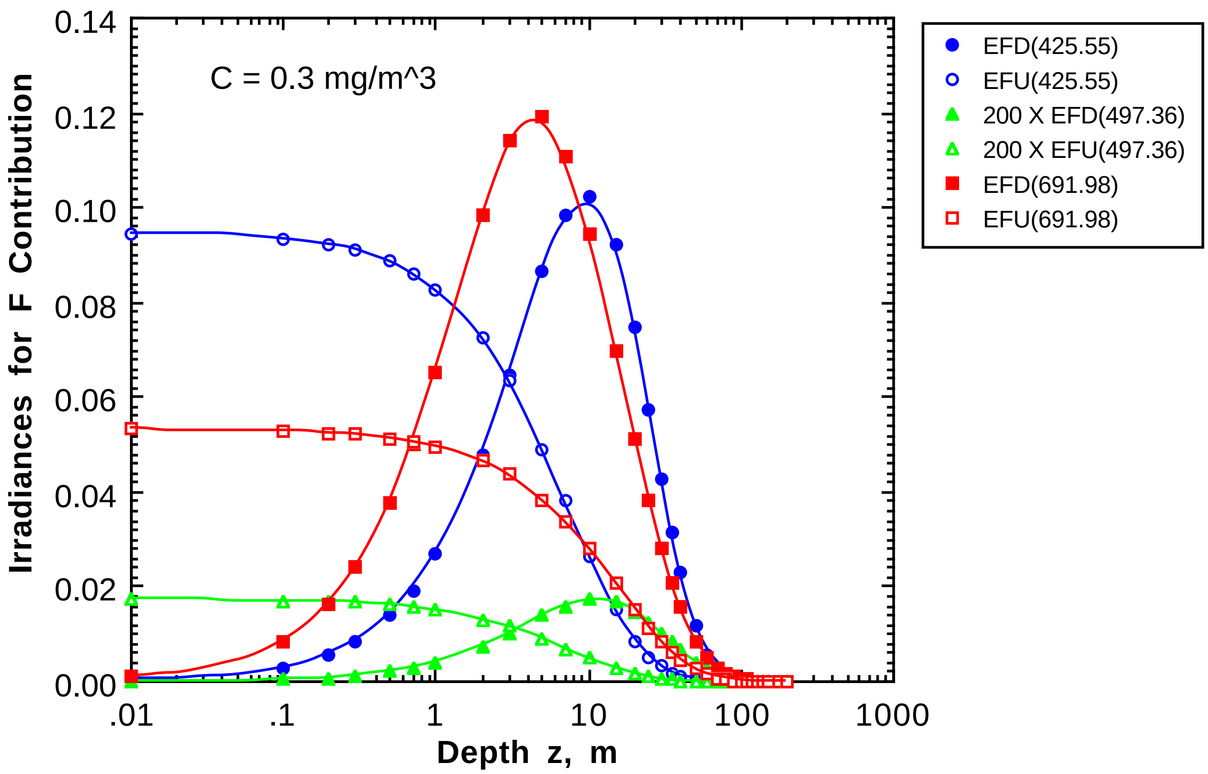

Figure 23. Depth dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in medium–productivity water (C=0.3 mg/m^3) at different wavelengths.

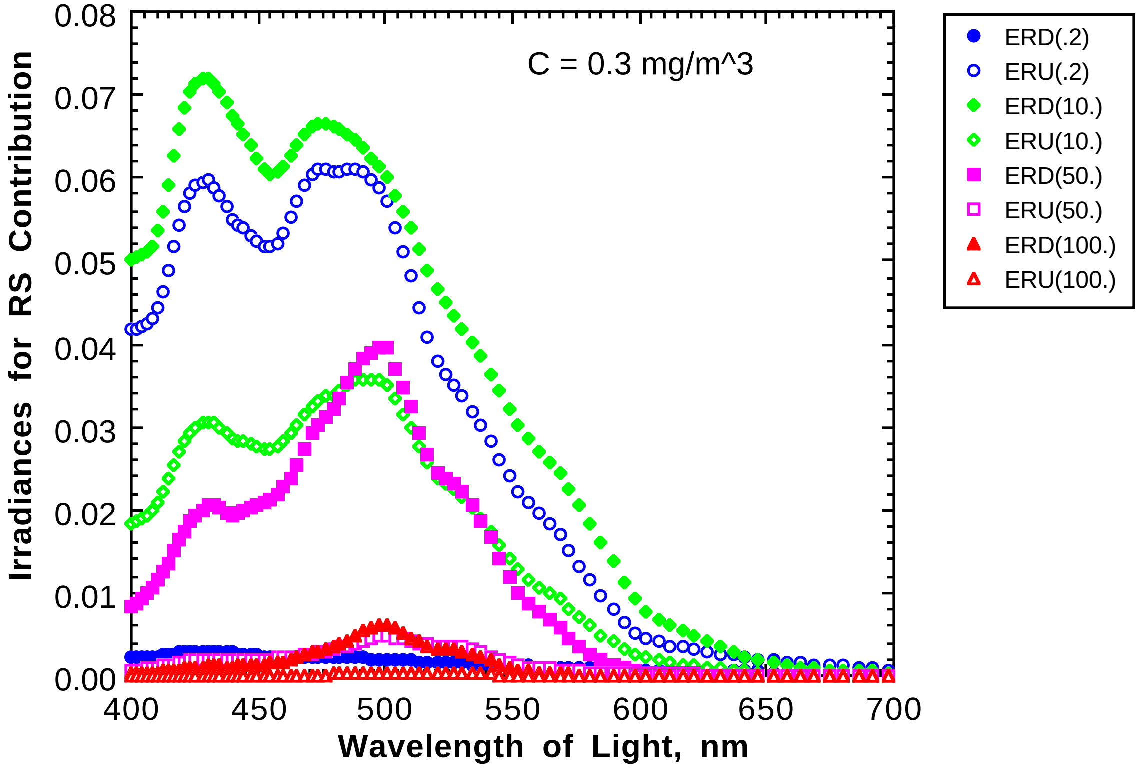

Figure 24. Spectral dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in medium–productivity water (C=0.3 *mg/m*^{3}) at different depths.

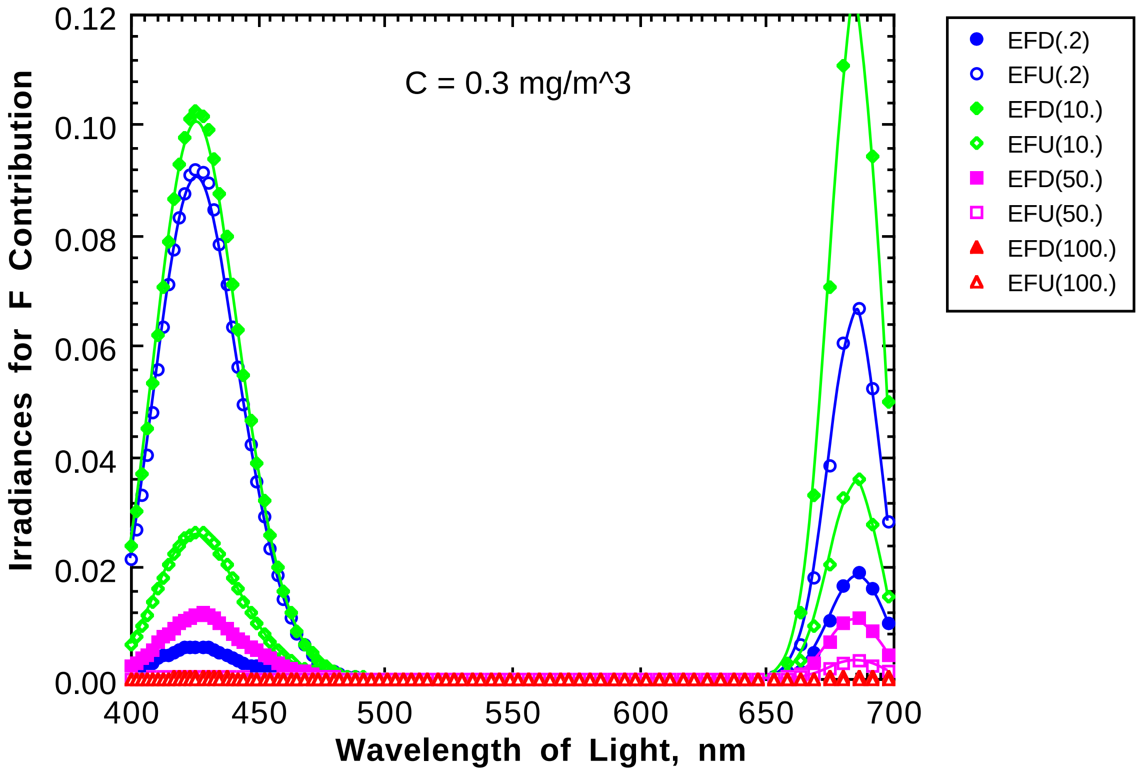

Figure 25. Spectral dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in medium–productivity water (C=0.3 *mg/m*^{3}) at different depths.

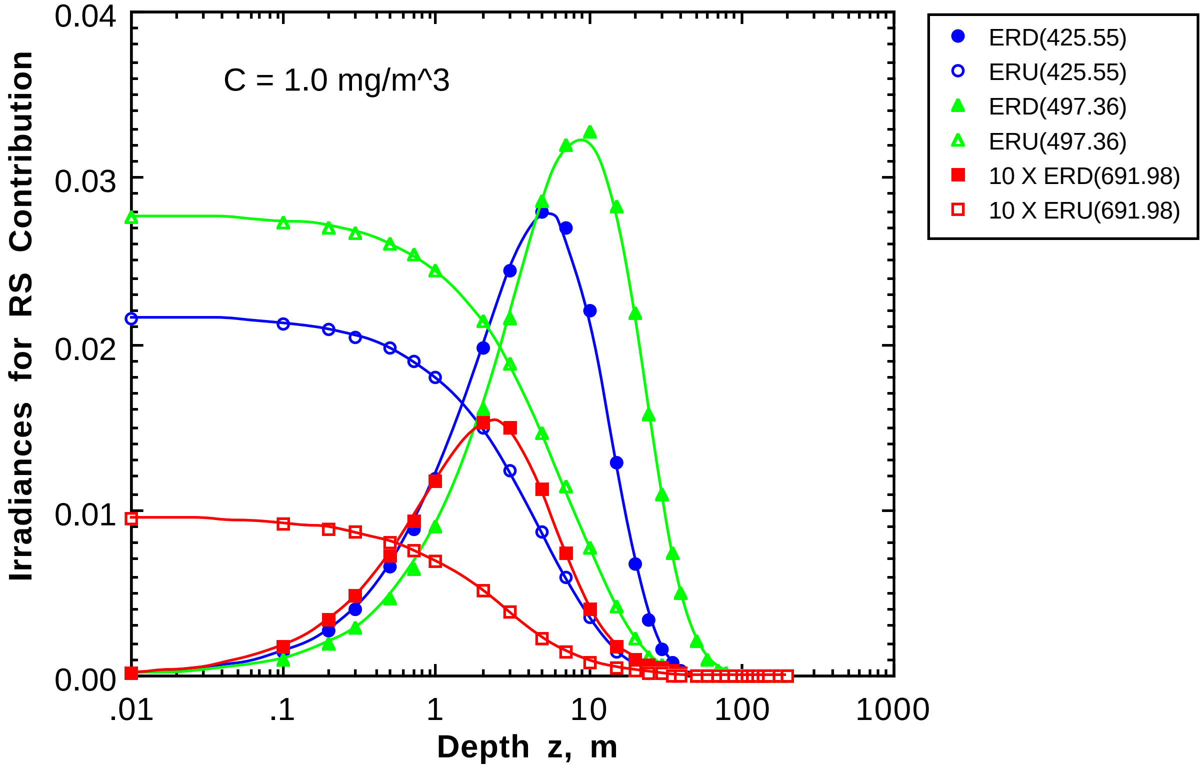

Figure 26. Depth dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in high–productivity water (C=1.0 mg/m^3) at different wavelengths.

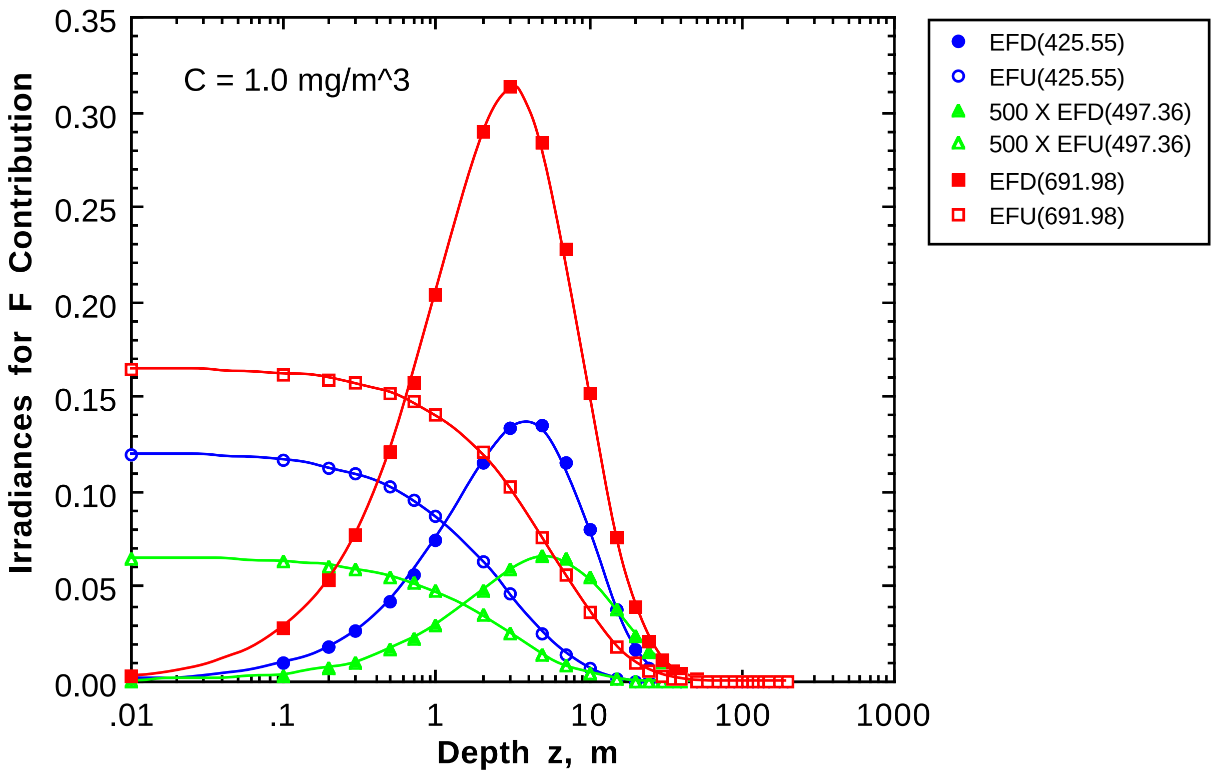

Figure 27. Depth dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in high–productivity water (C=1.0 mg/m^3) at different wavelengths.

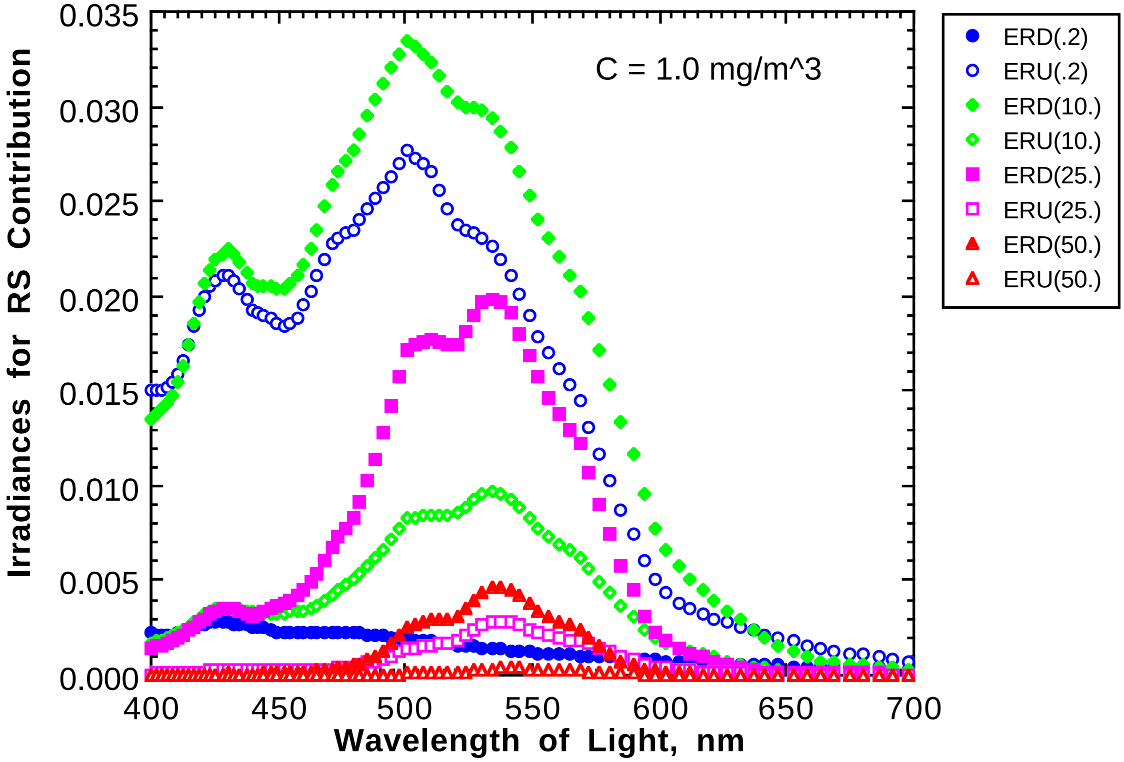

Figure 28. Spectral dependence of Downward (ERD) and Upward (ERU) Irradiances for Raman Scattering Contribution in high–productivity water (C=1.0 mg/m^3) at different depths.

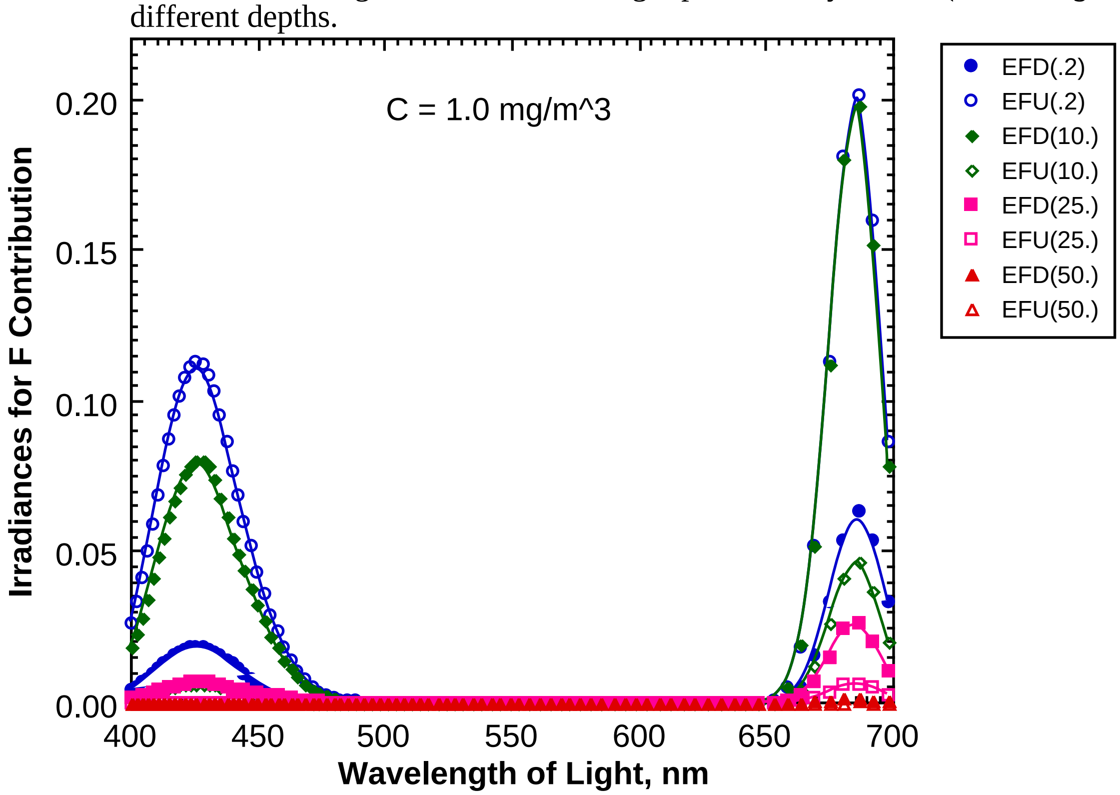

Figure 29. Spectral dependence of Downward (EFD) and Upward (EFU) Irradiances for Fluorescent Contribution in high–productivity water (C=1.0 mg/m^3) at different depths.

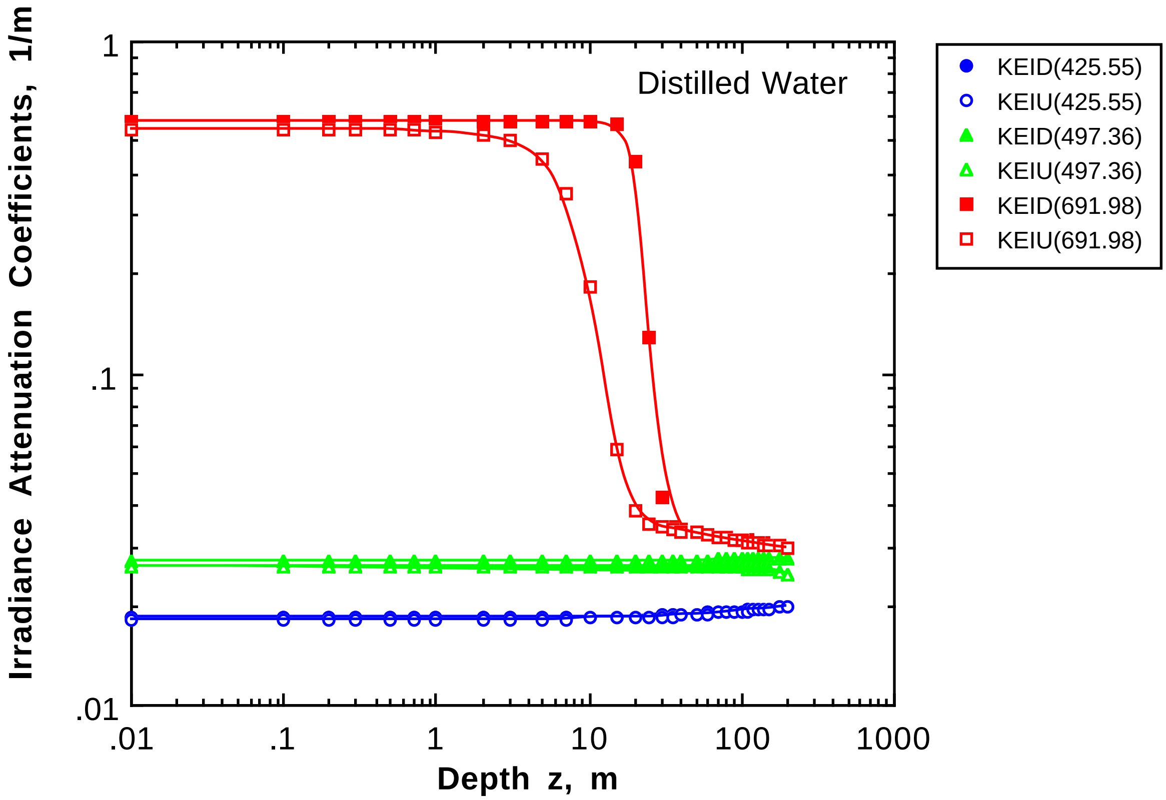

Figure 30. Depth dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in distilled water at different wavelengths.

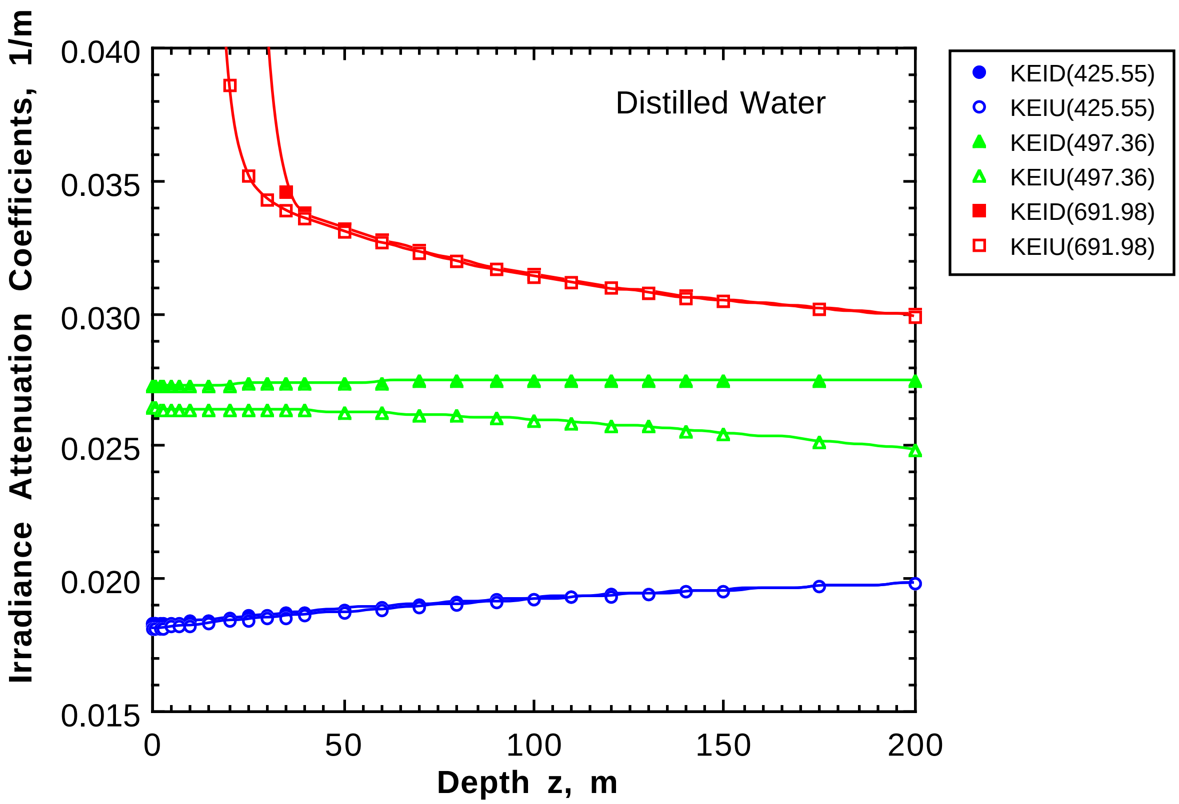

Figure 31. The same as Figure 30 but with magnified display (Convergence of KEID(497.36) and KEIU(497.36) occur at depths greater than 200 *m*.

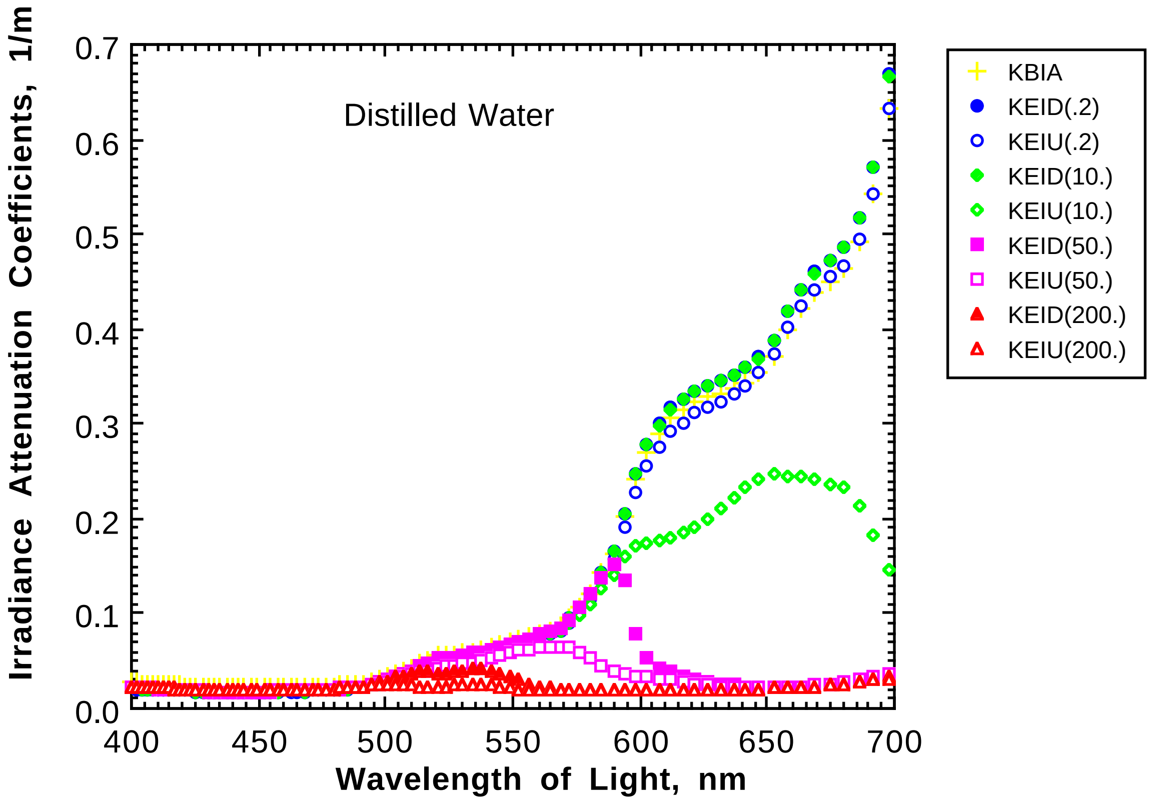

Figure 32. Spectral dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in distilled water at different depths.

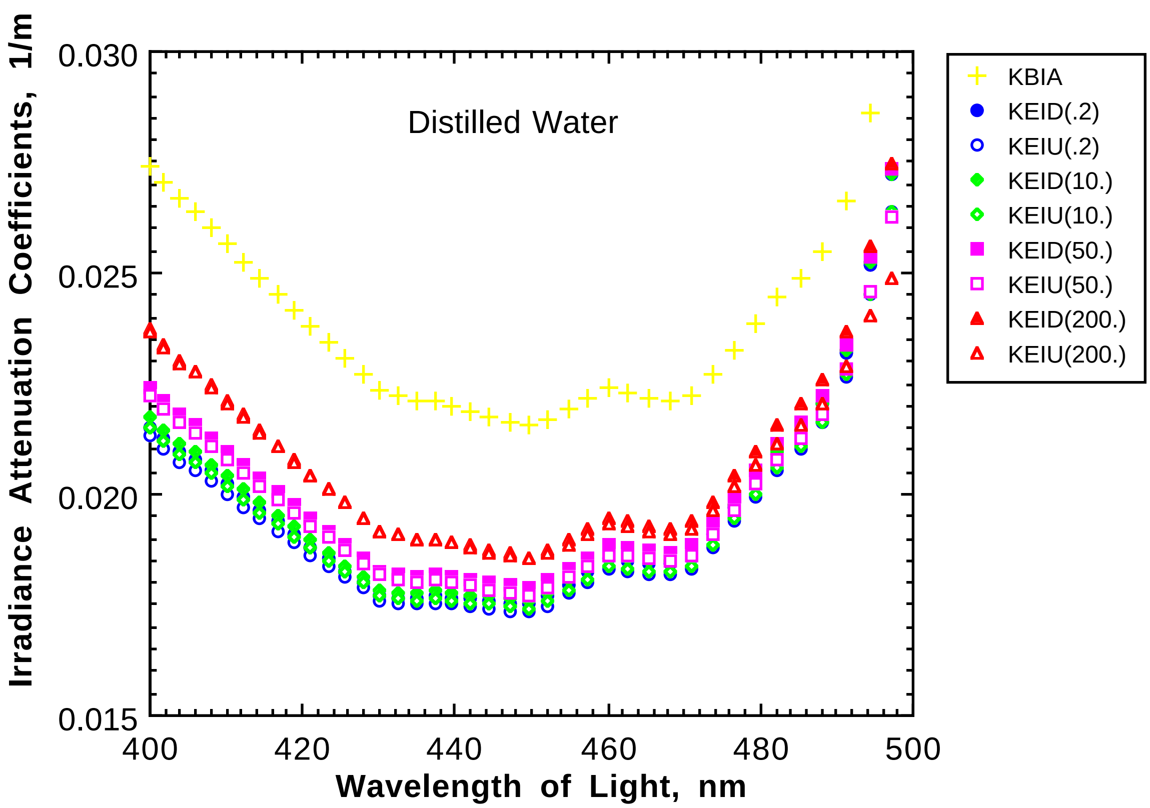

Figure 33. The same as Figure 32 but with magnified display in blue–green part of spectrum.

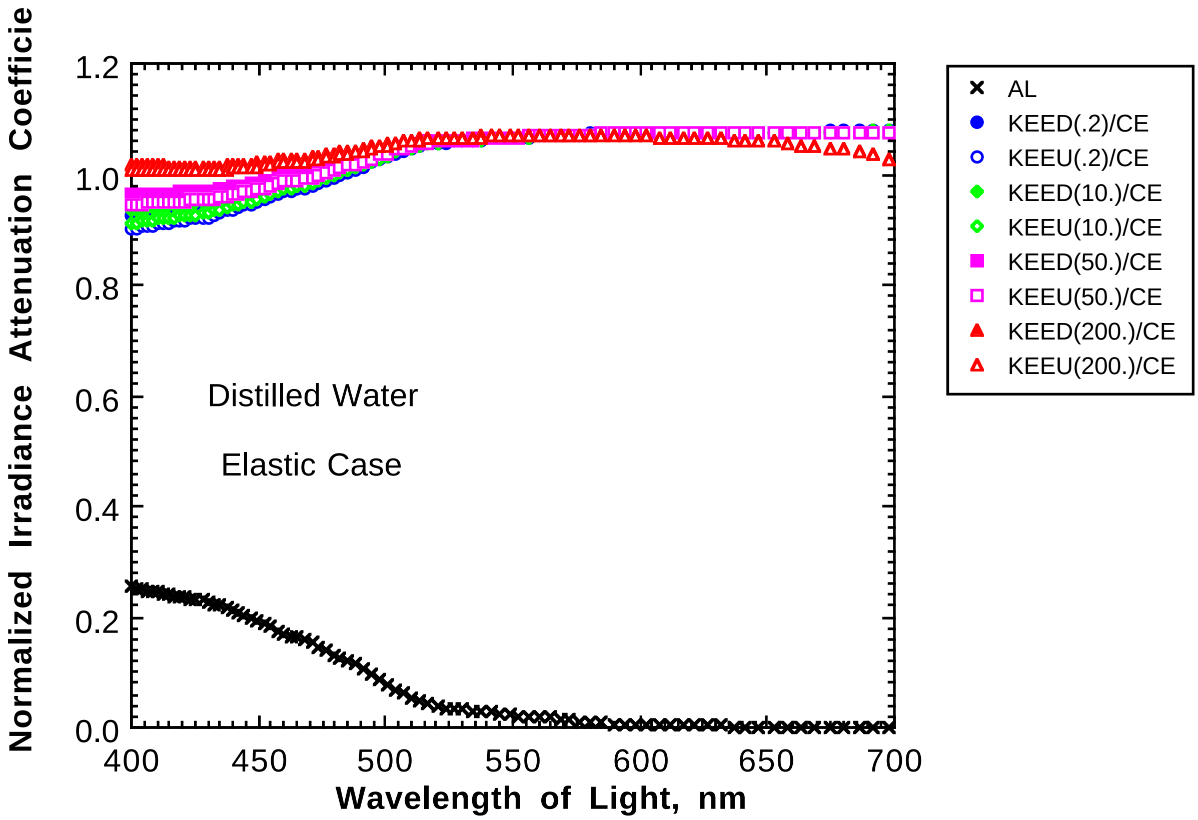

Figure 34. Spectral dependence of Normalized (by extinction coefficient $c^E = a + b^E$) Irradiance Attenuation Coefficients in distilled water at different depths (Inelastic effects are turned off).

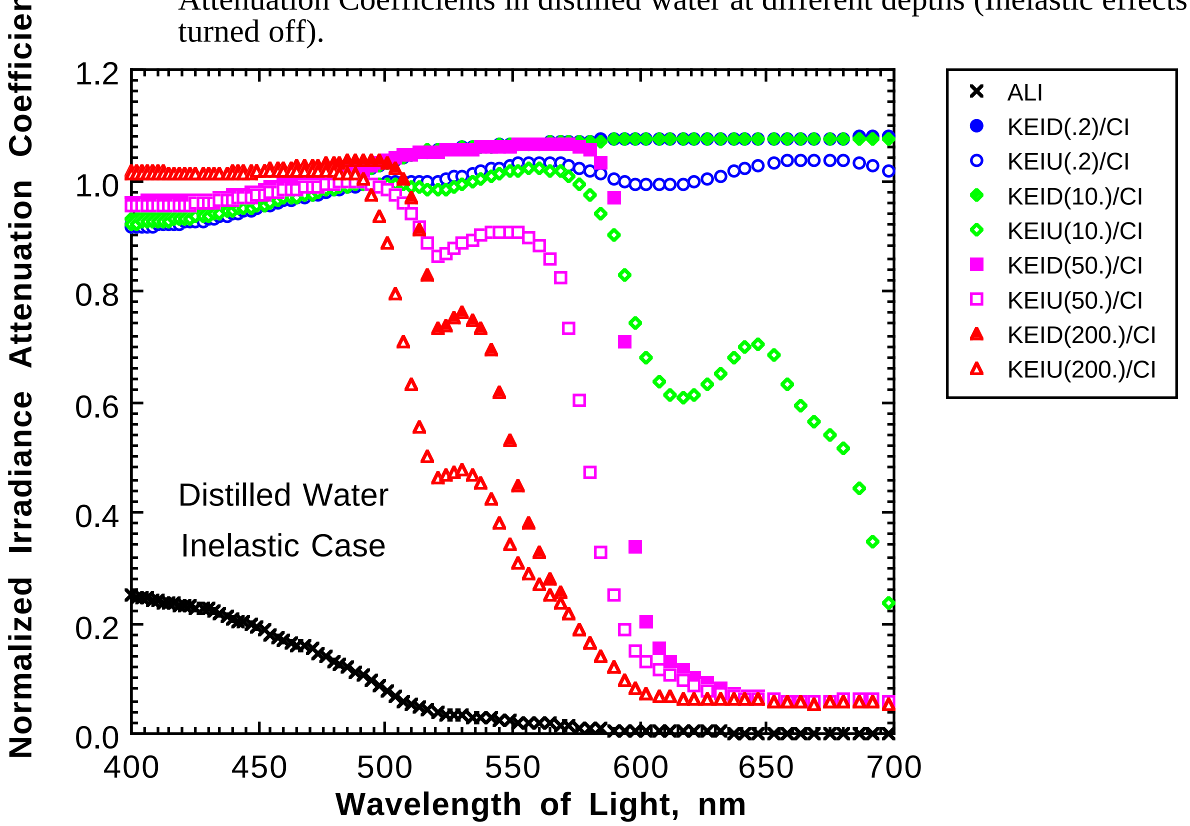

Figure 35. Spectral dependence of Normalized (by extinction coefficient $c = a + b$) Irradiance Attenuation Coefficients in distilled water at different depths (Inelastic effects are taken into account).

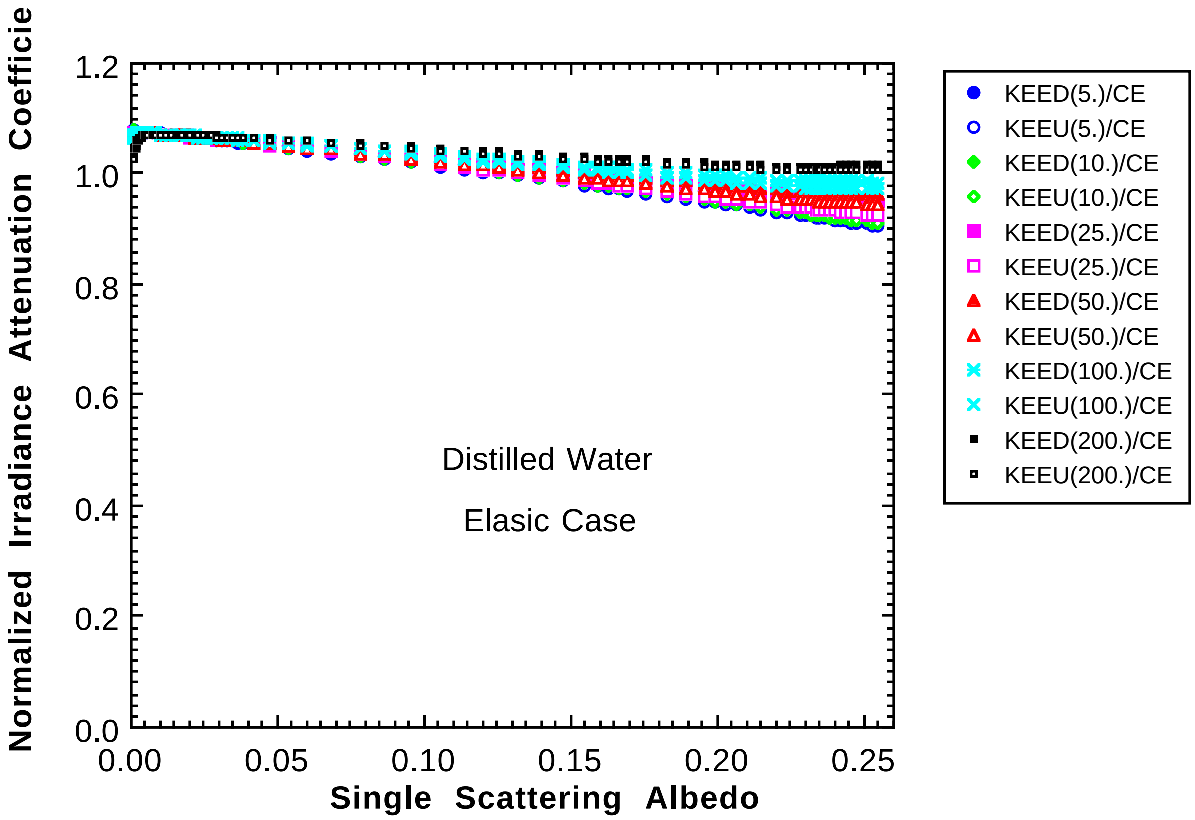

Figure 36. Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in distilled water at different depths as a function of single-scattering albedo (Inelastic effects are turned off).

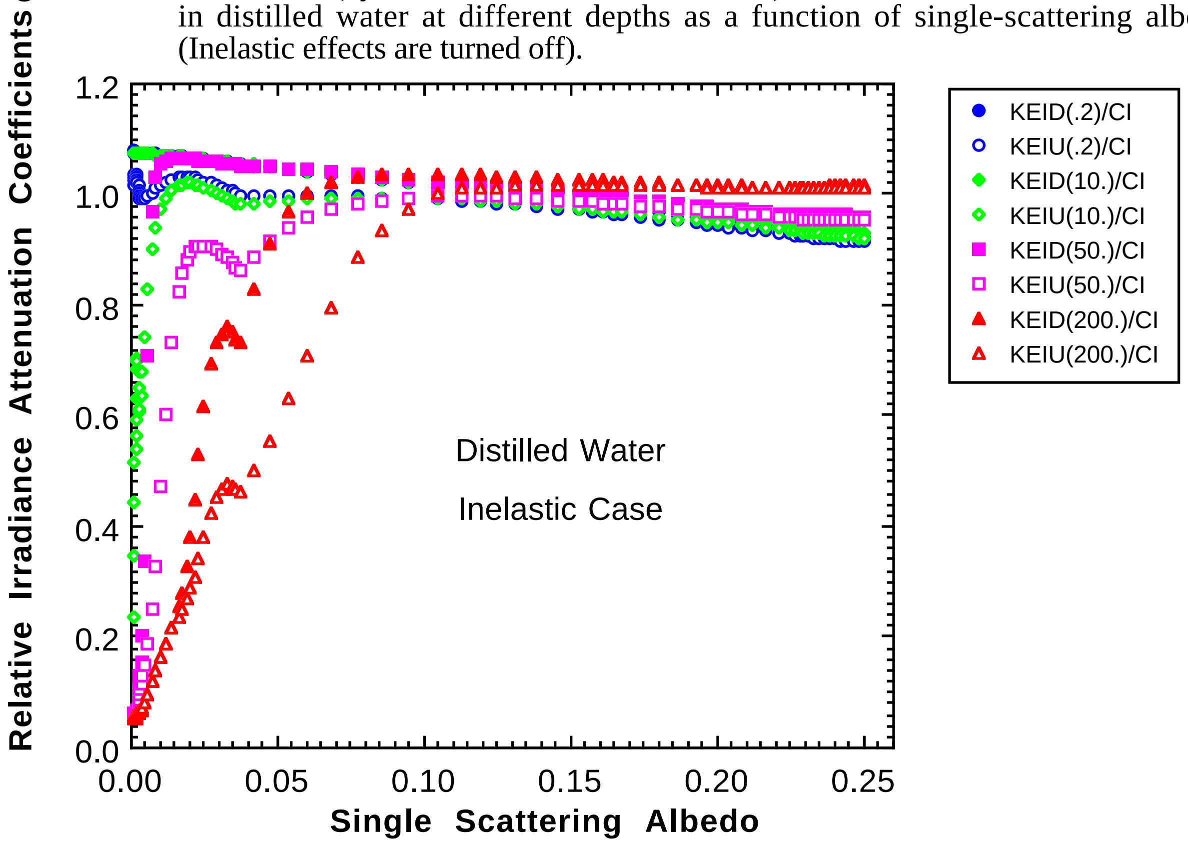

Figure 37. Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in distilled water at different depths as a function of single-scattering albedo (Inelastic effects are taken into account).

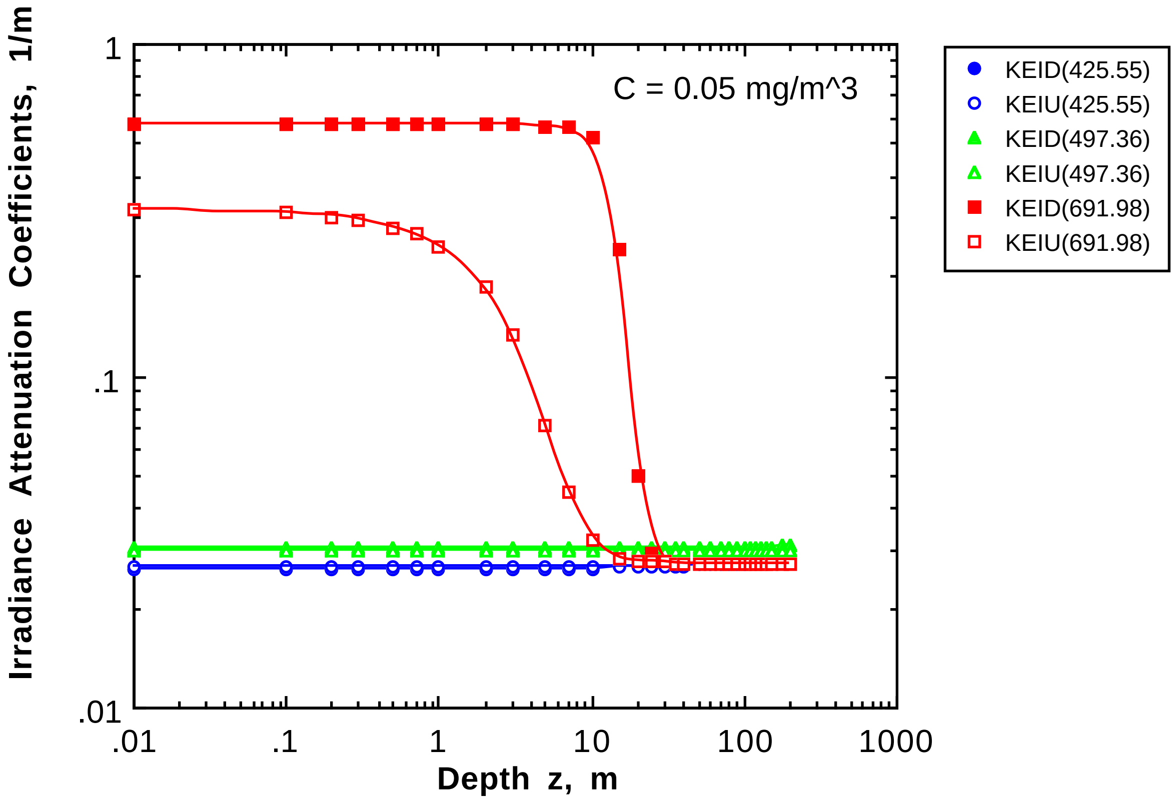

Figure 38. Depth dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in low-productivity water at different wavelengths.

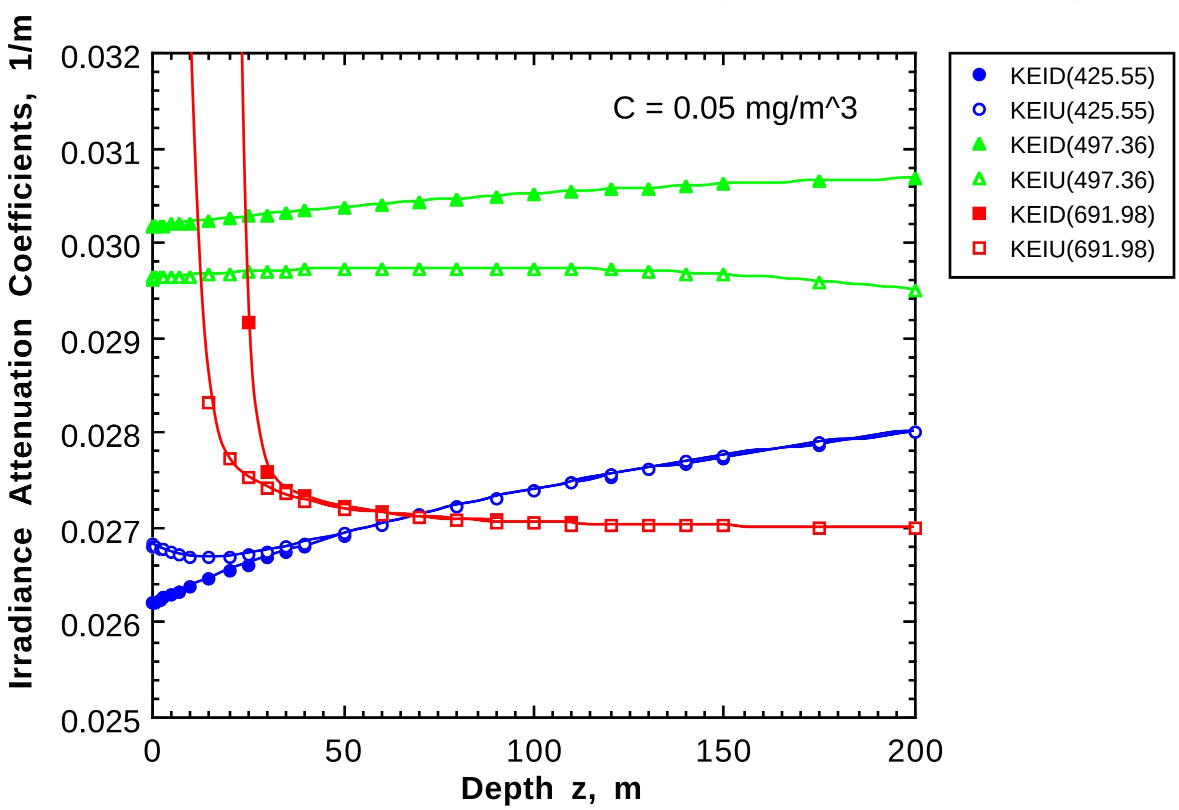

Figure 39. The same as Figure 38 but with magnified display (Convergence of KEID(497.36) and KEIU(497.36) occur at depths greater than 200 *m*.

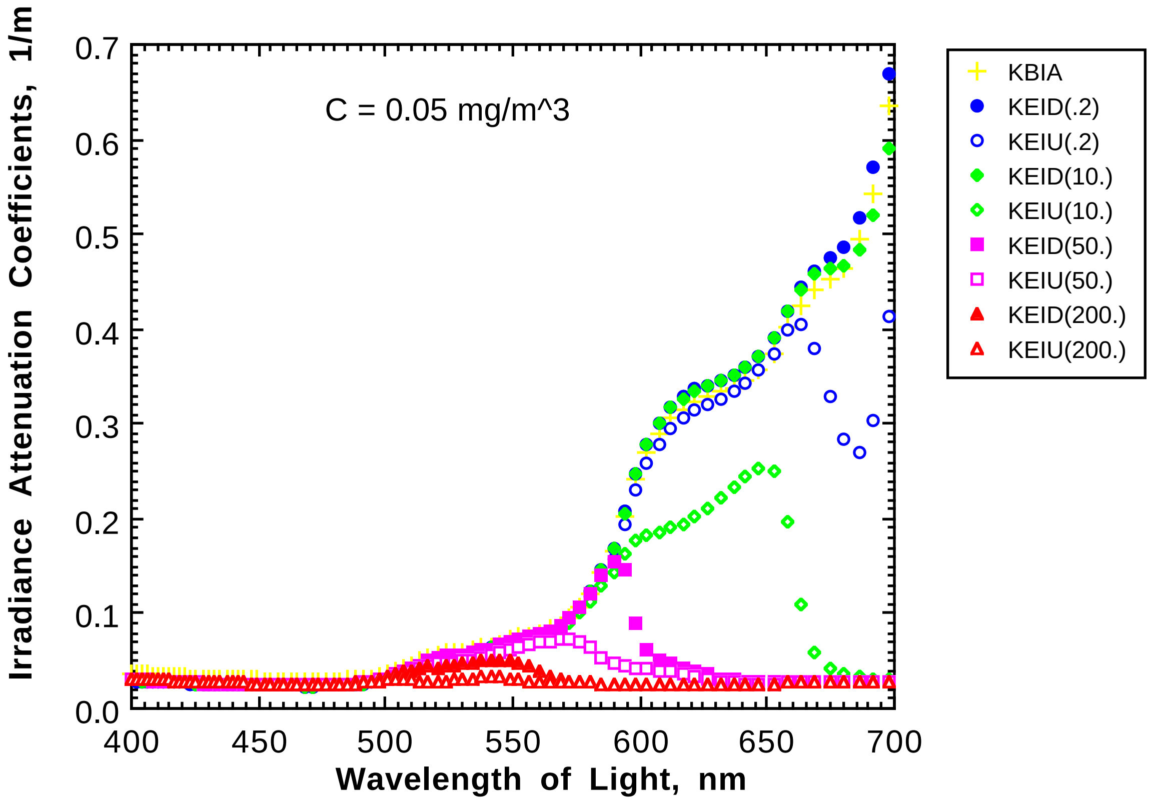

Figure 40. Spectral dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in low-productivity water at different depths.

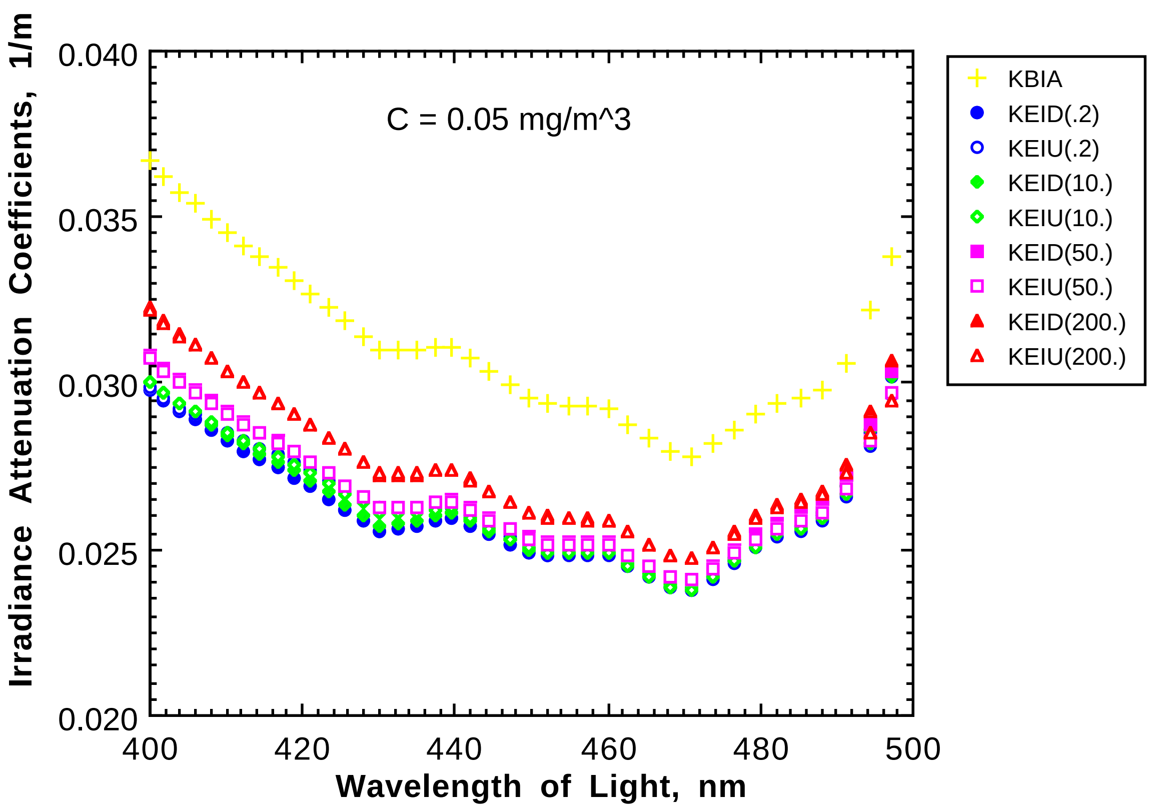

Figure 41. The same as Figure 40 but with magnified display in blue–green part of spectrum.

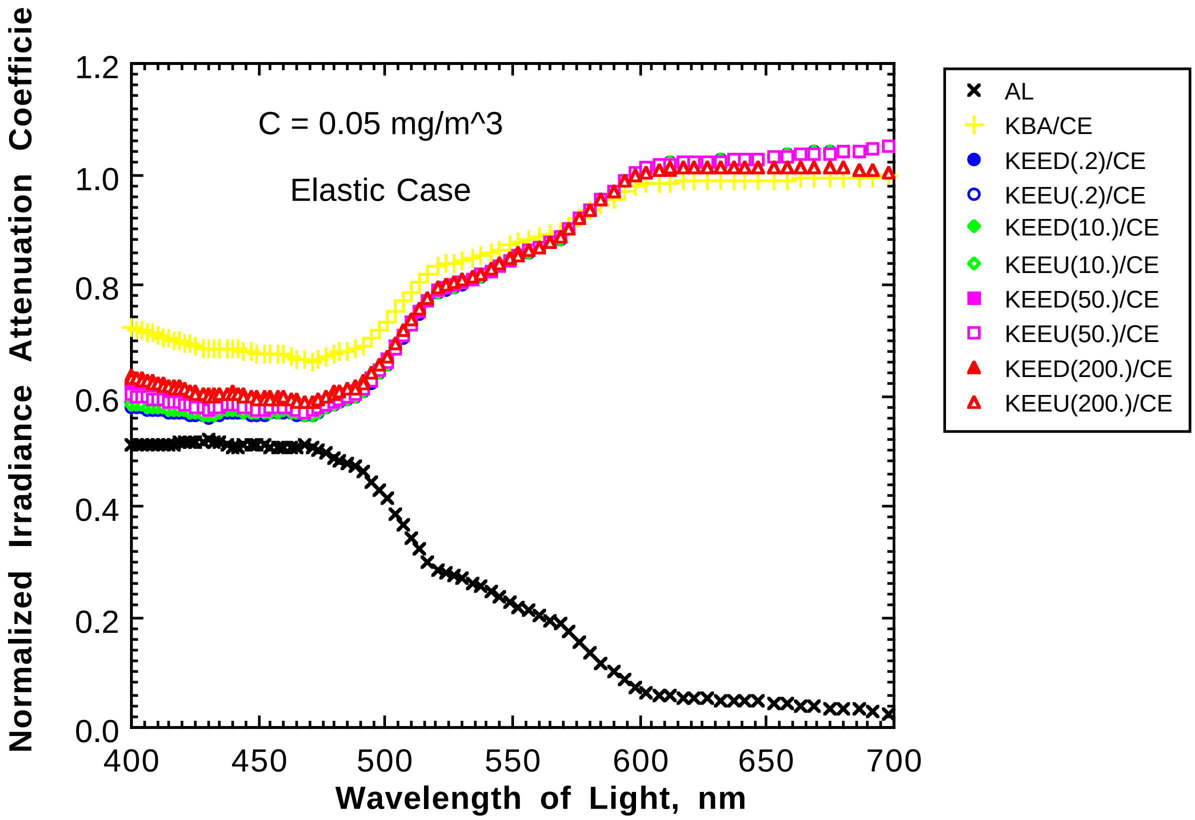

Figure 42. Spectral dependence of Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in low-productivity water at different depths (Inelastic effects are turned off).

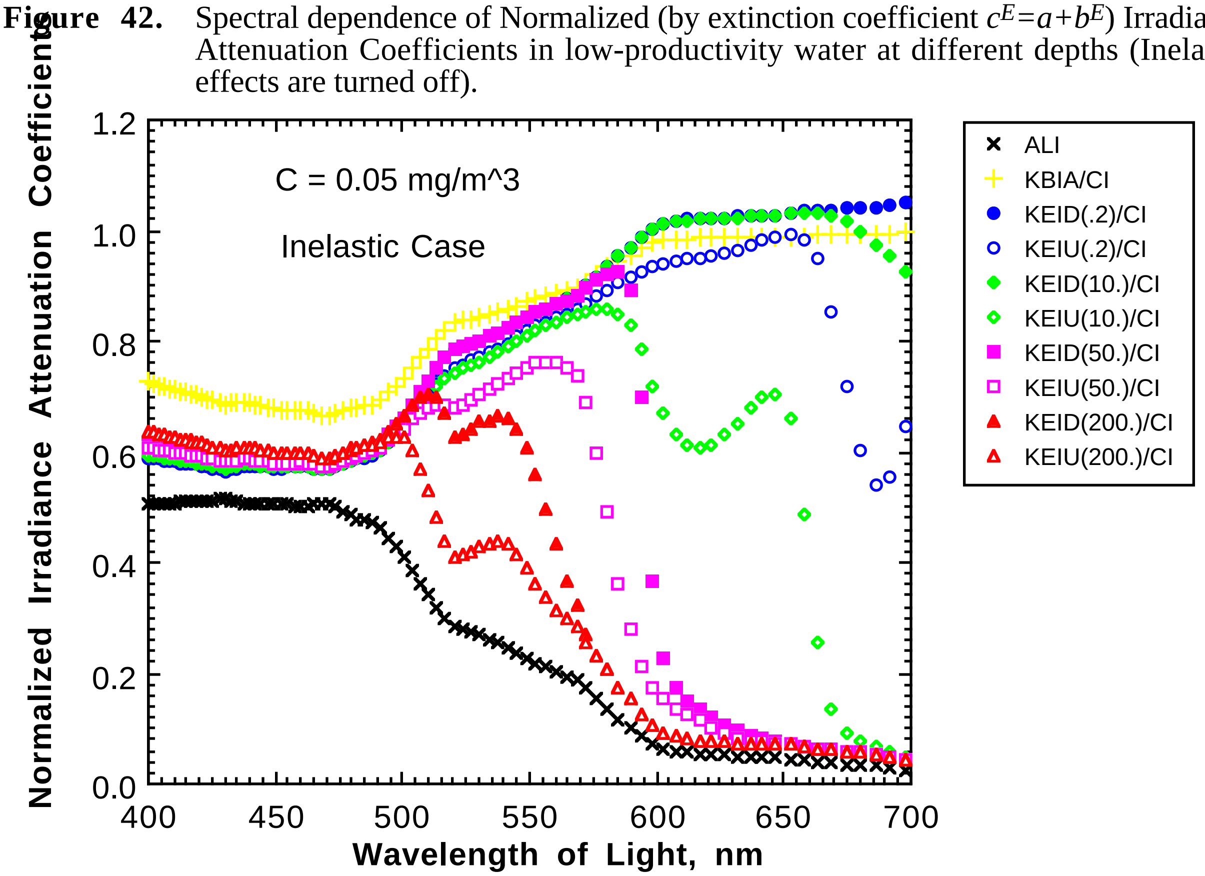

Figure 43. Spectral dependence of Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in low-productivity water at different depths (Inelastic effects are taken into account).

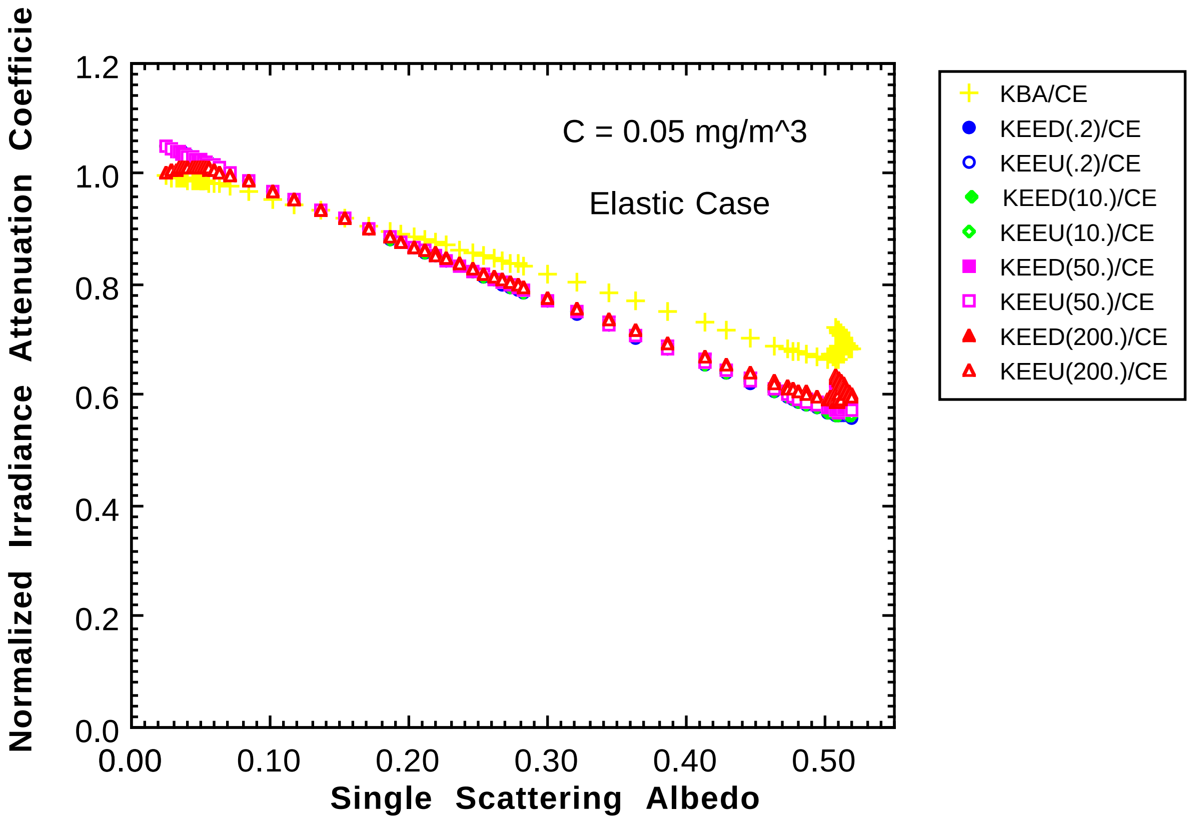

Figure 44. Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in low-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are turned off).

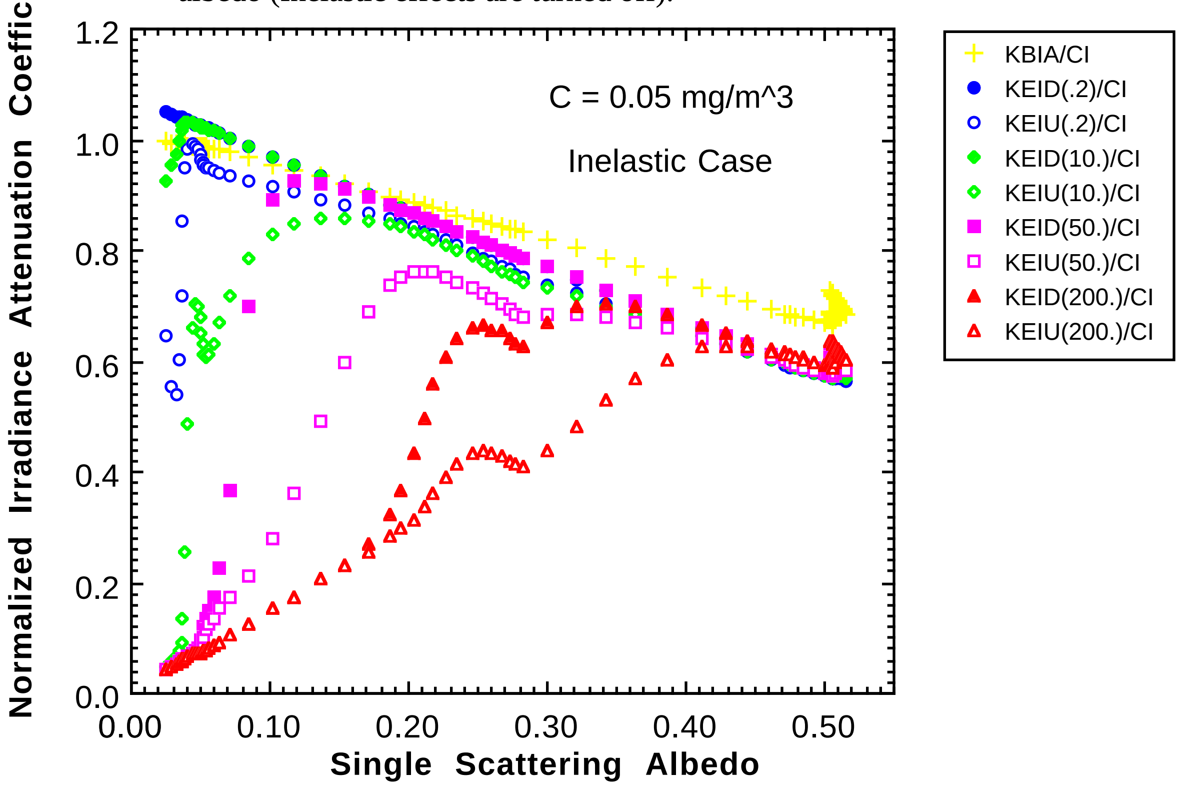

Figure 45. Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in low-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are taken into account).

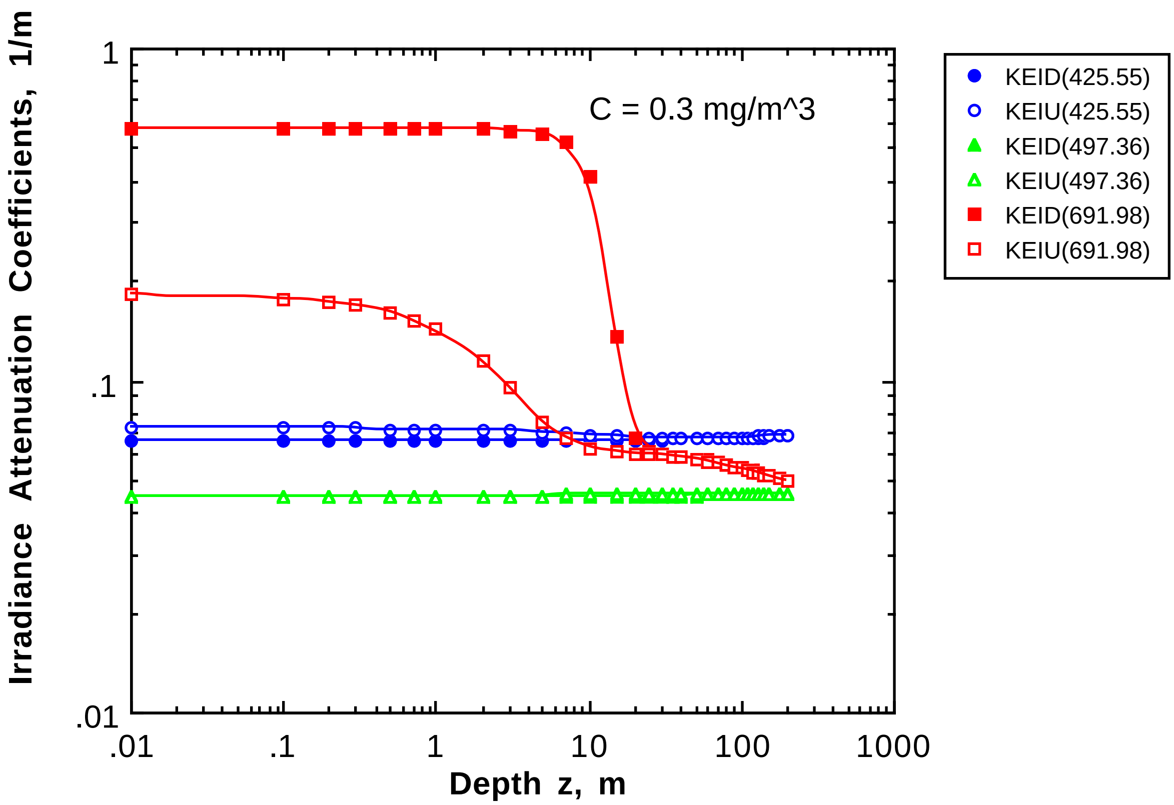

Figure 46. Depth dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in medium-productivity water at different wavelengths.

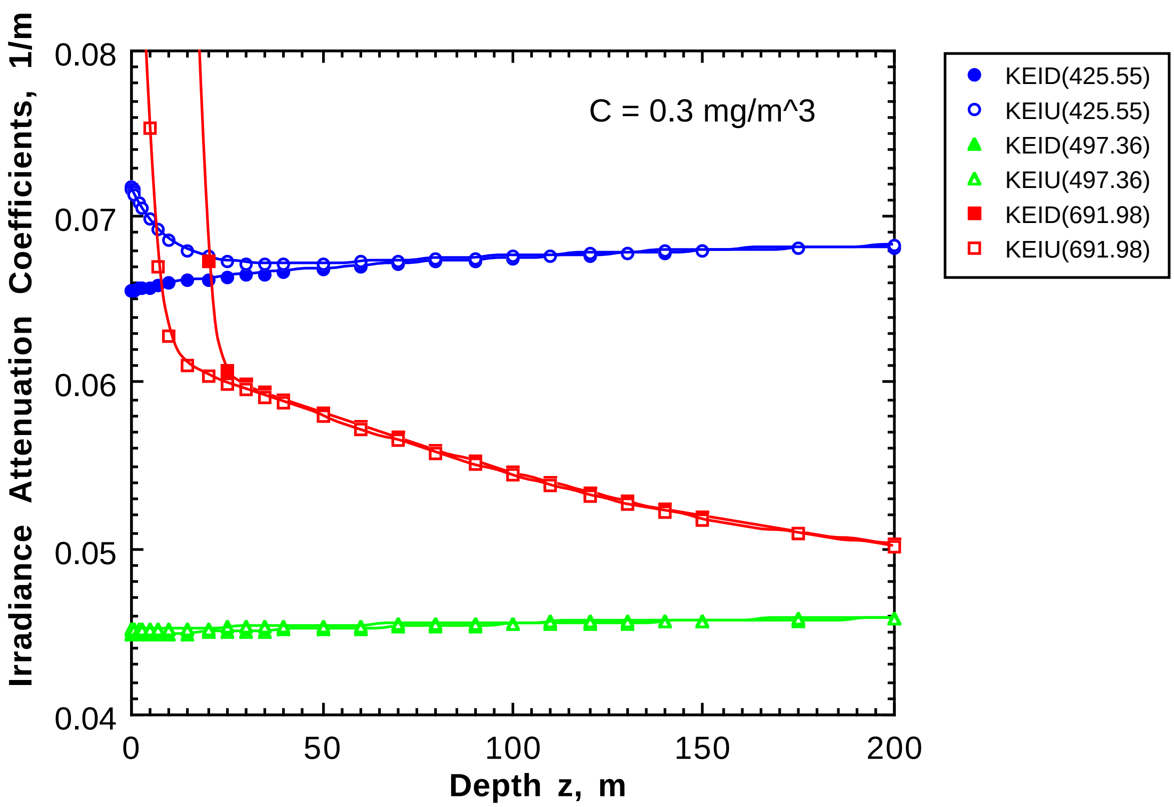

Figure 47. The same as Figure 46 but with magnified display.

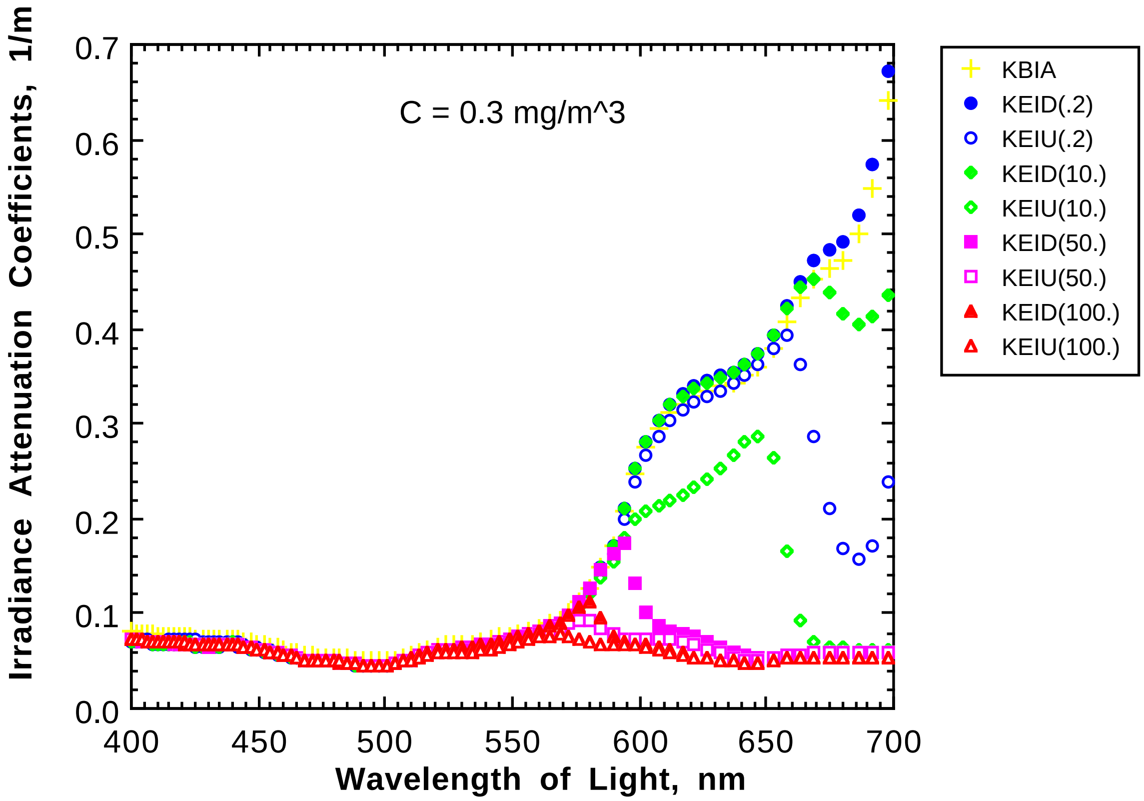

Figure 48. Spectral dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in medium-productivity water at different depths.

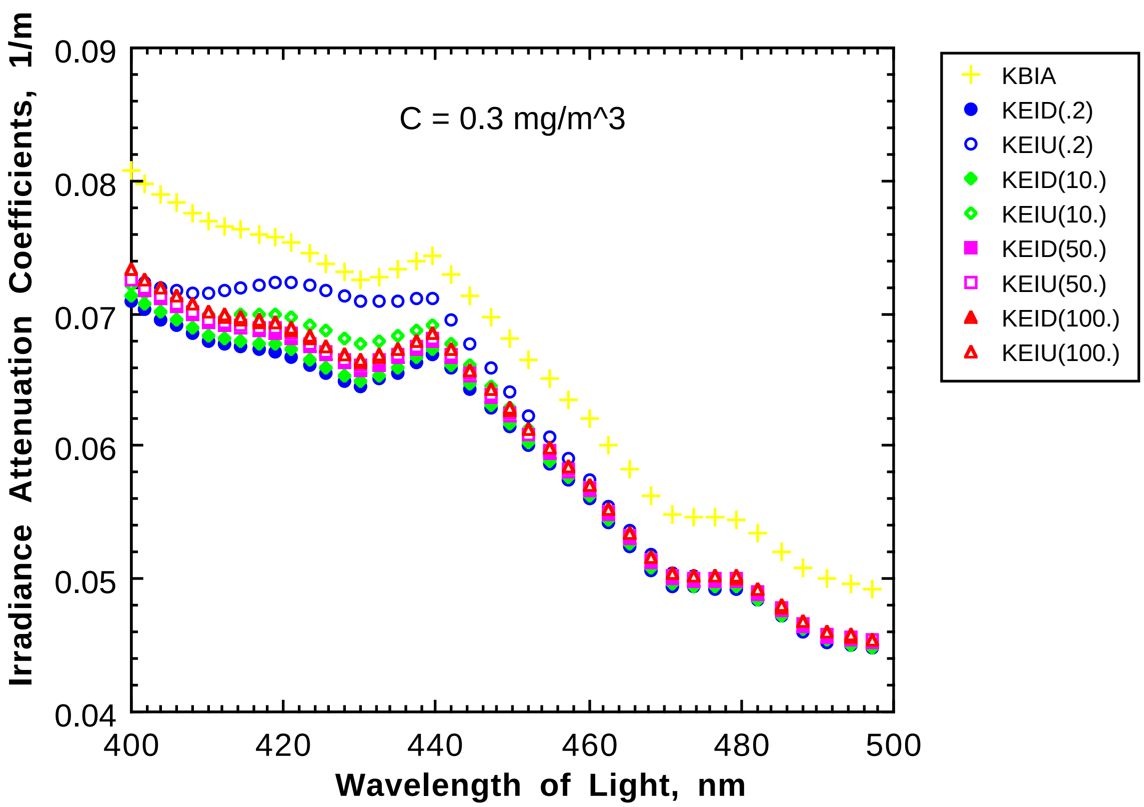

Figure 49. The same as Figure 48 but with magnified display in the blue–green part of spectrum.

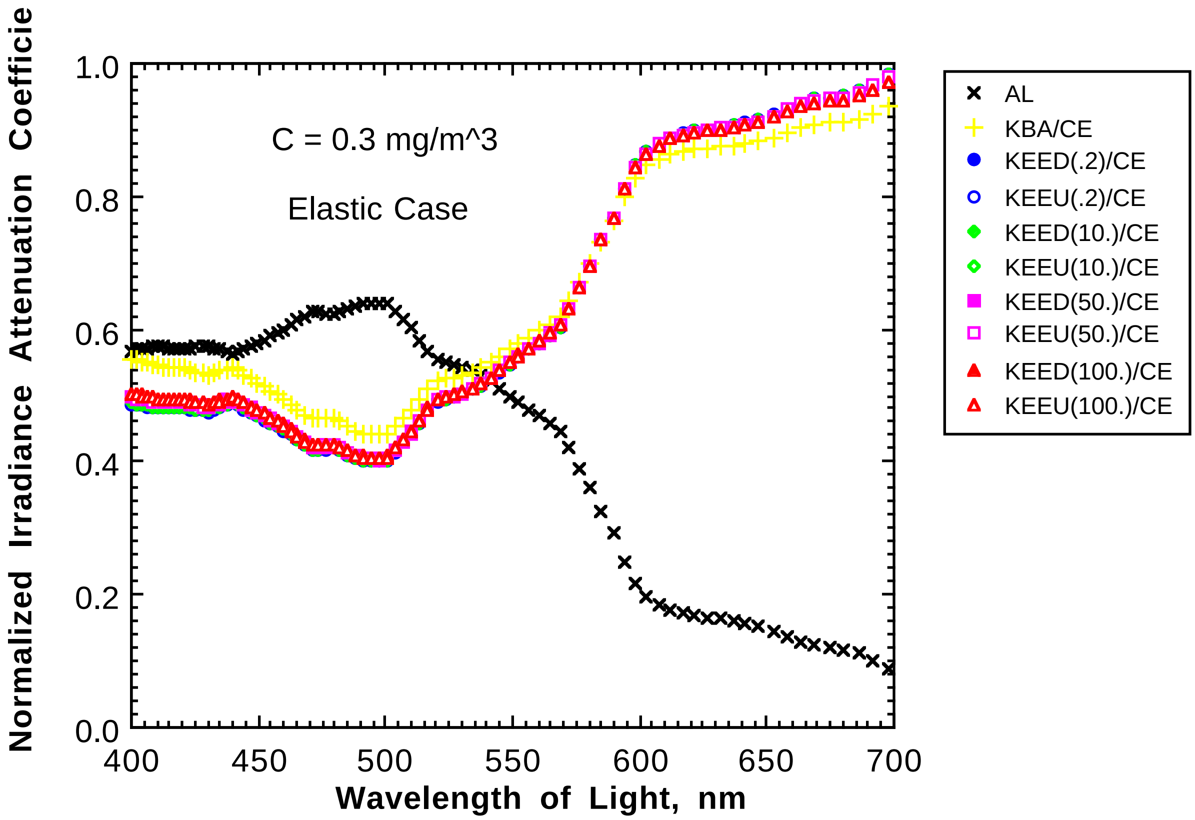

Figure 50. Spectral dependence of Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in medium-productivity water at different depths (Inelastic effects are turned off).

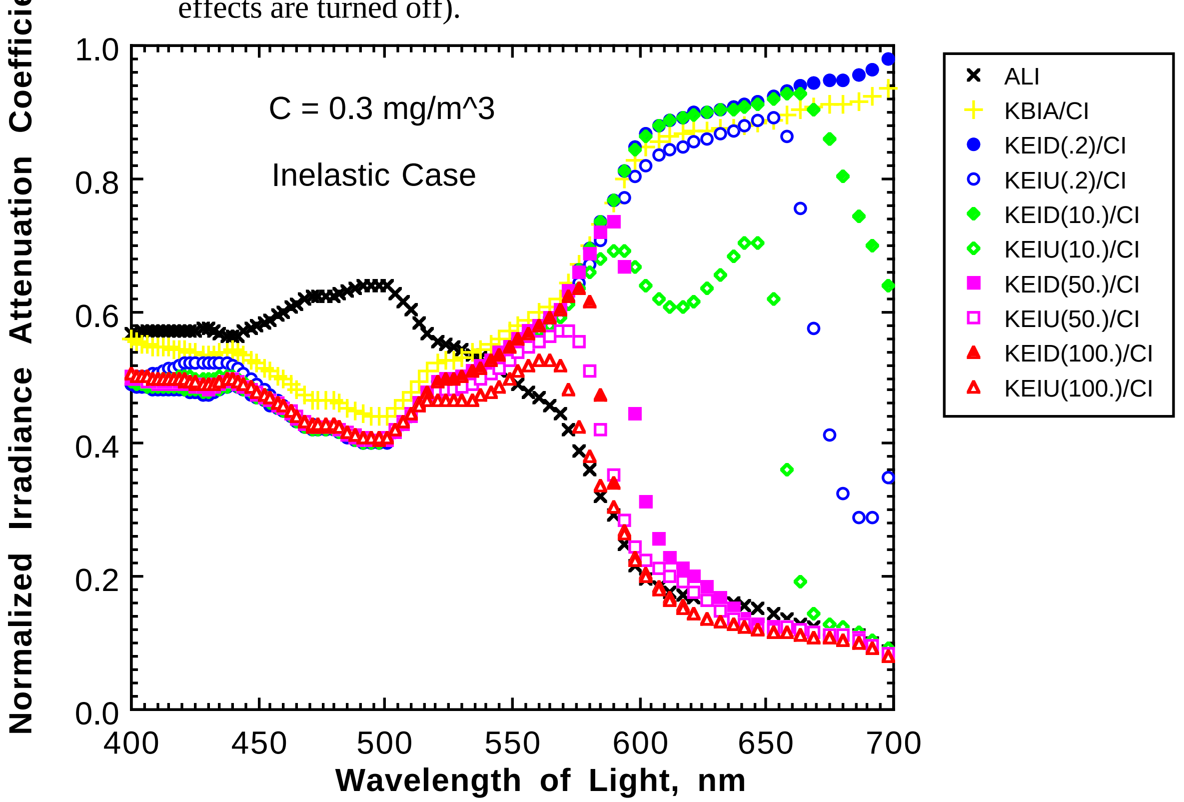

Figure 51. Spectral dependence of Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in medium-productivity water at different depths (Inelastic effects are taken into account).

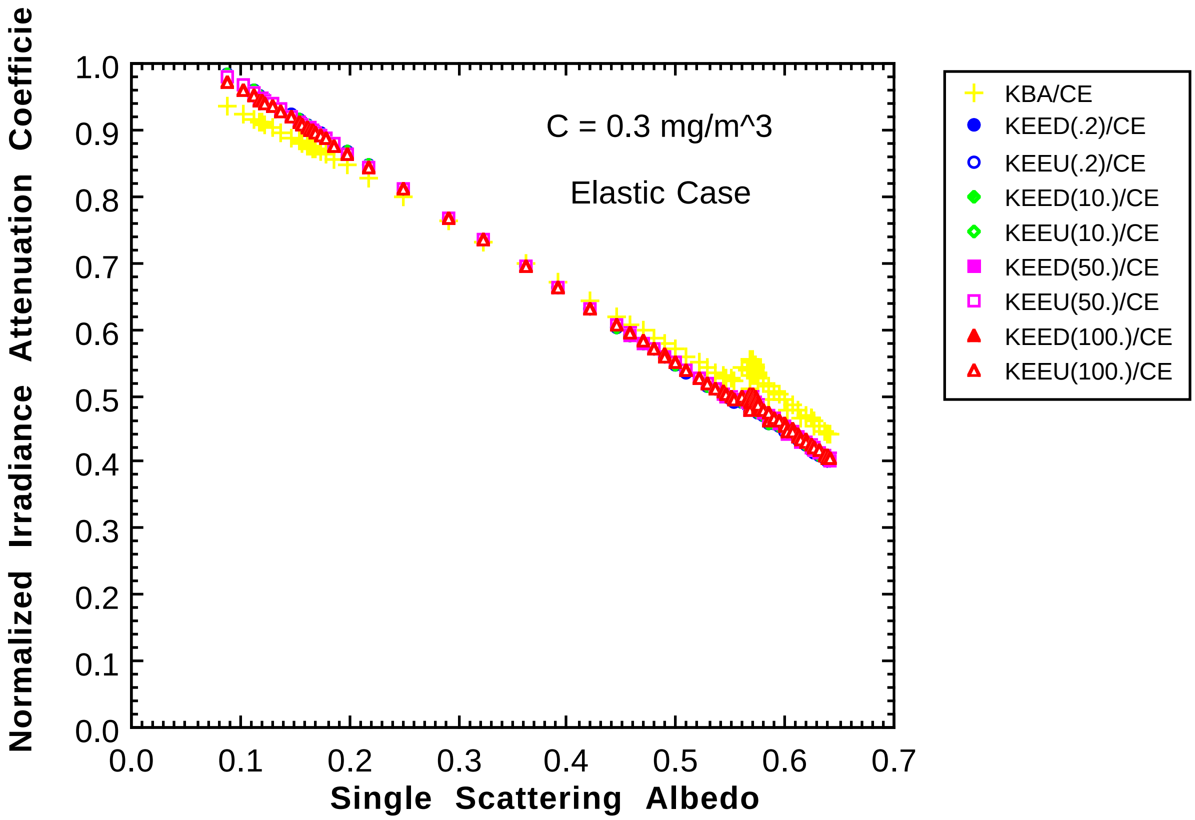

Figure 52. Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in medium-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are turned off).

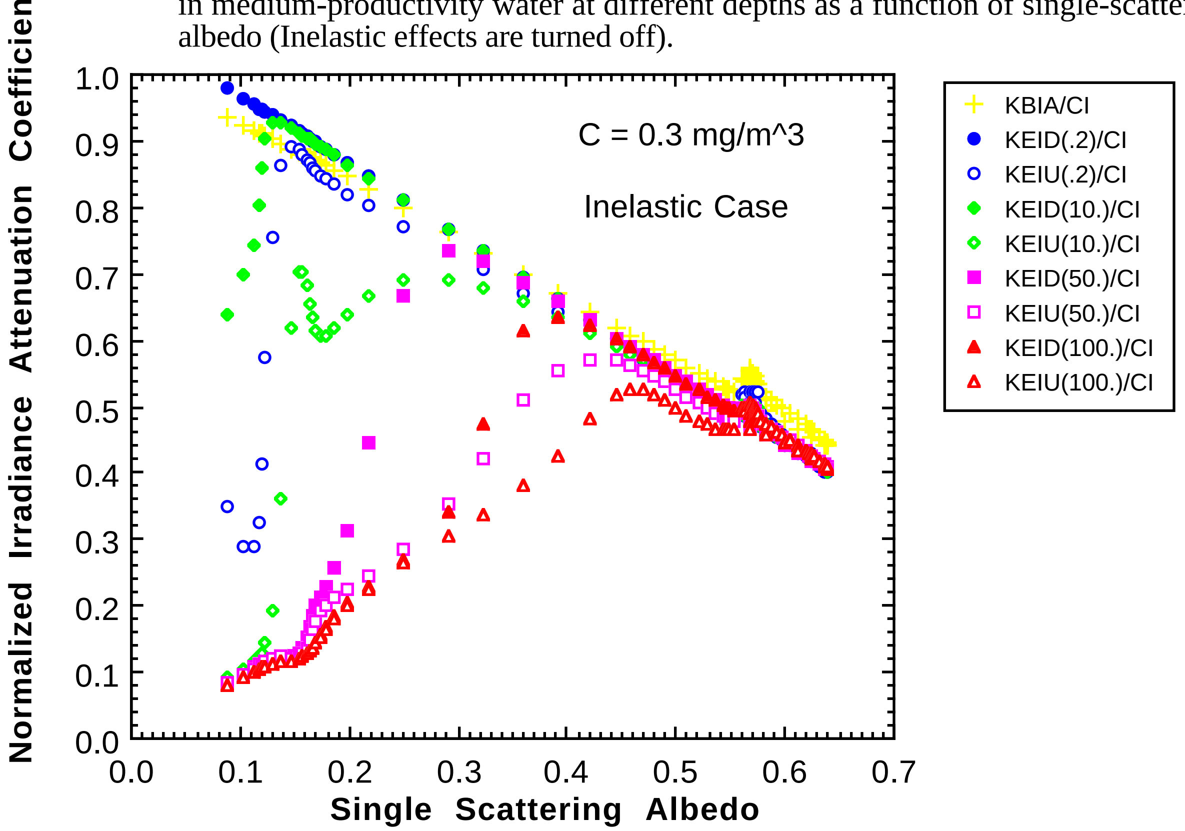

Figure 53. Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in medium-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are taken into account).

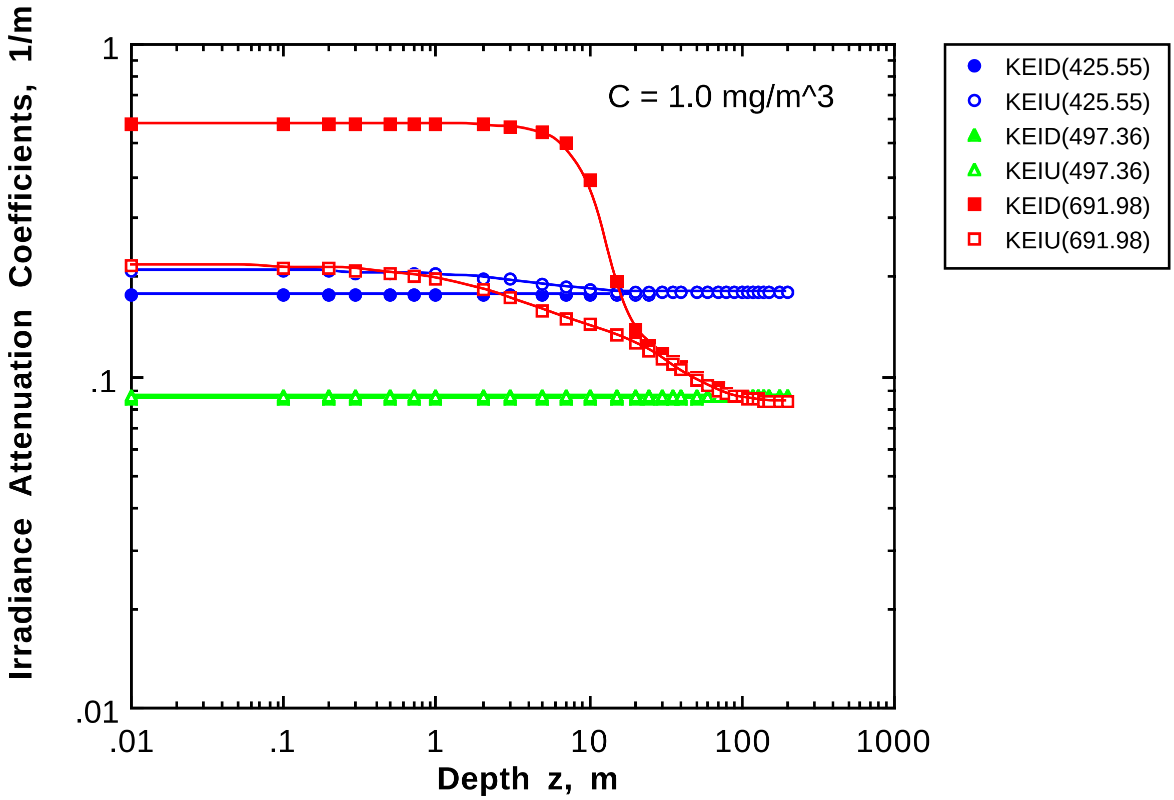

Figure 54. Depth dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in high-productivity water at different wavelengths.

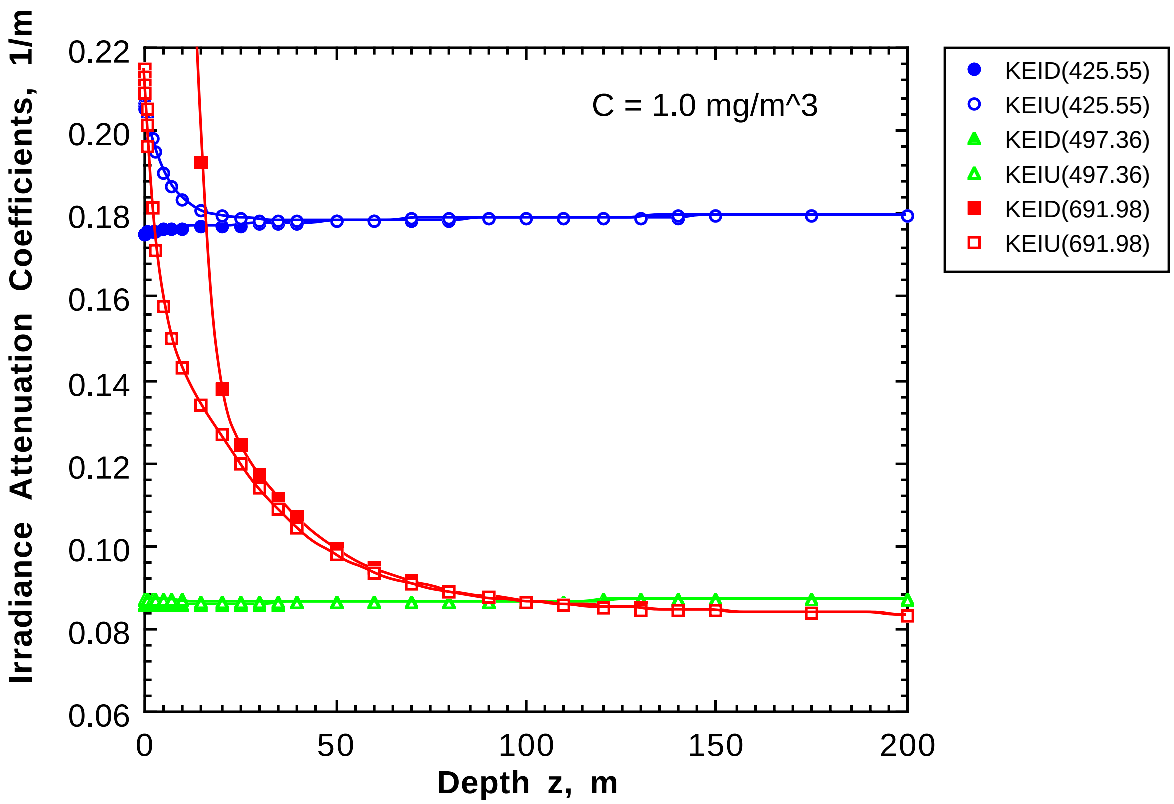

Figure 55. The same as Figure 54 but with magnified display.

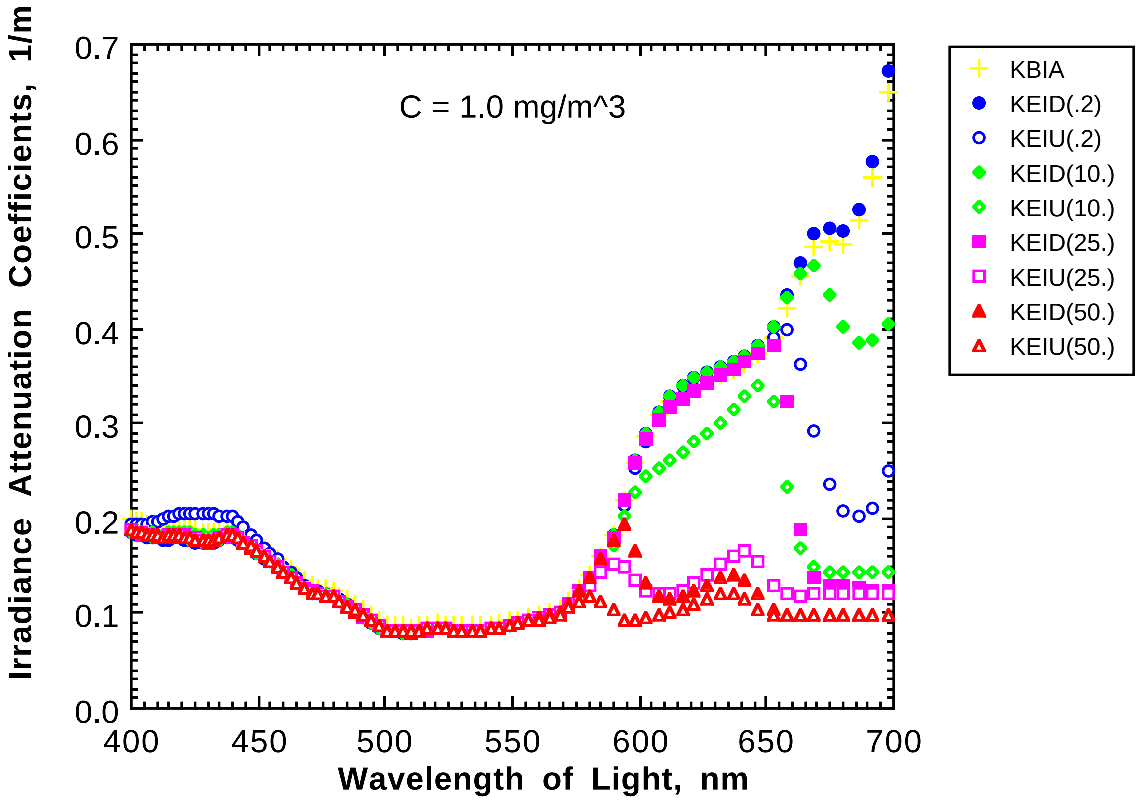

Figure 56. Spectral dependence of Downward (KEID) and Upward (KEIU) Irradiance Attenuation Coefficients in high-productivity water at different depths.

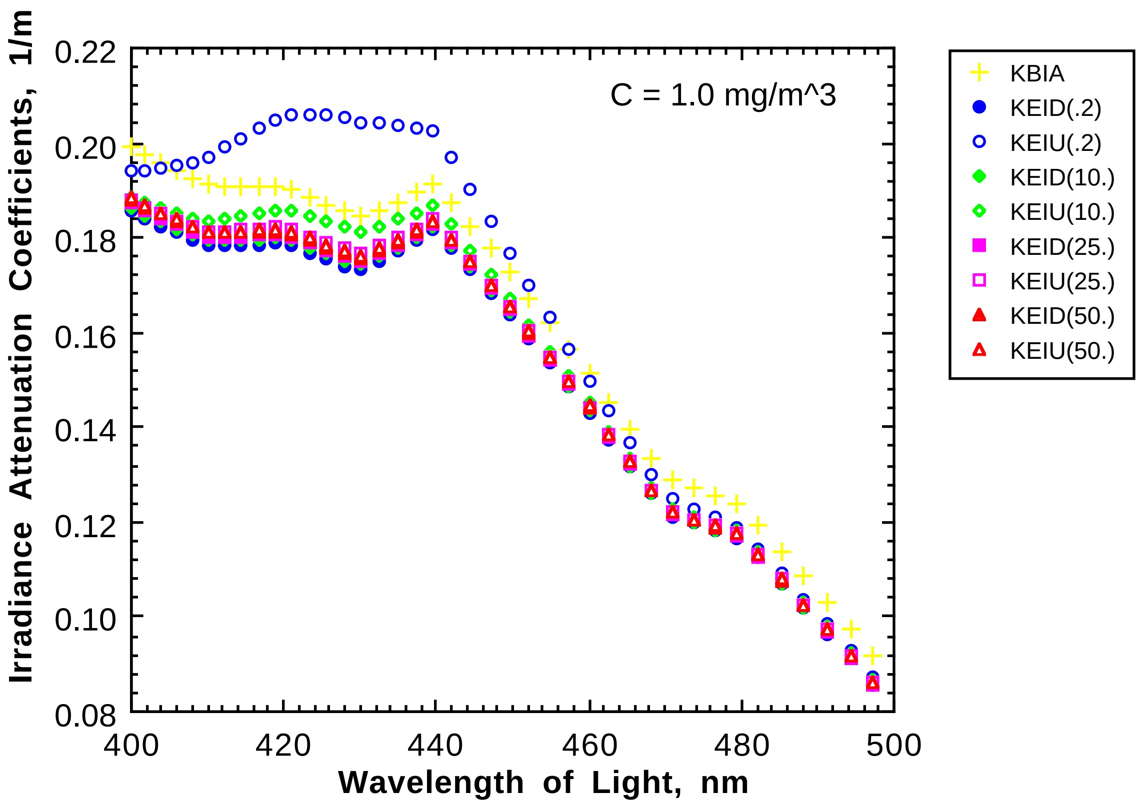

Figure 57. The same as Figure 56 but with magnified display in blue–green part of spectrum.

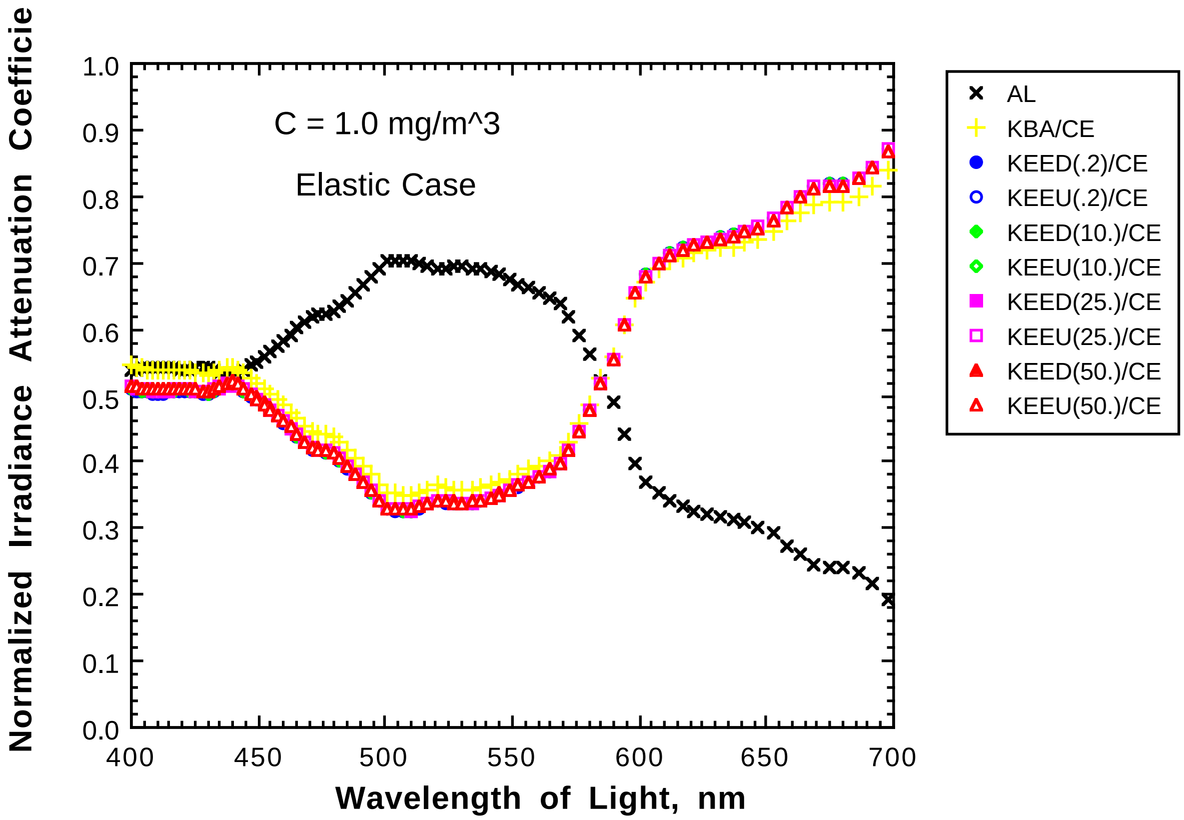

Figure 58. Spectral dependence of Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in high-productivity water at different depths (Inelastic effects are turned off).

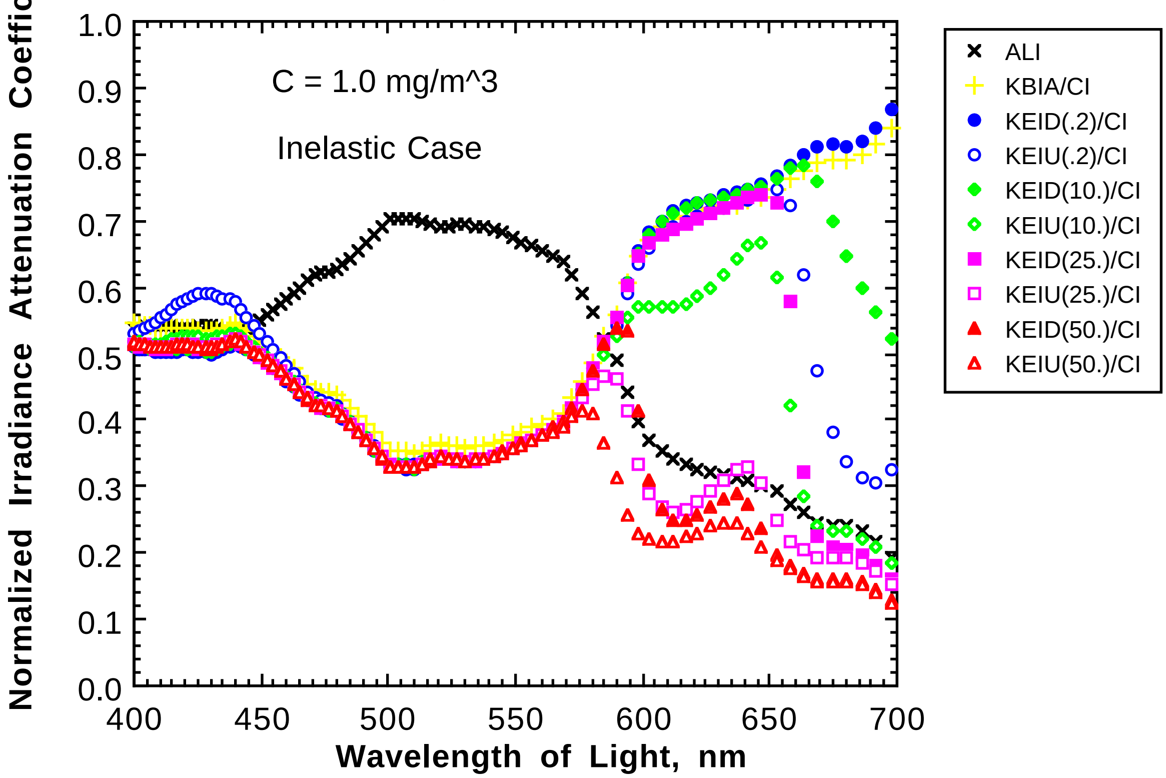

Figure 59. Spectral dependence of Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in high-productivity water at different depths (Inelastic effects are taken into account).

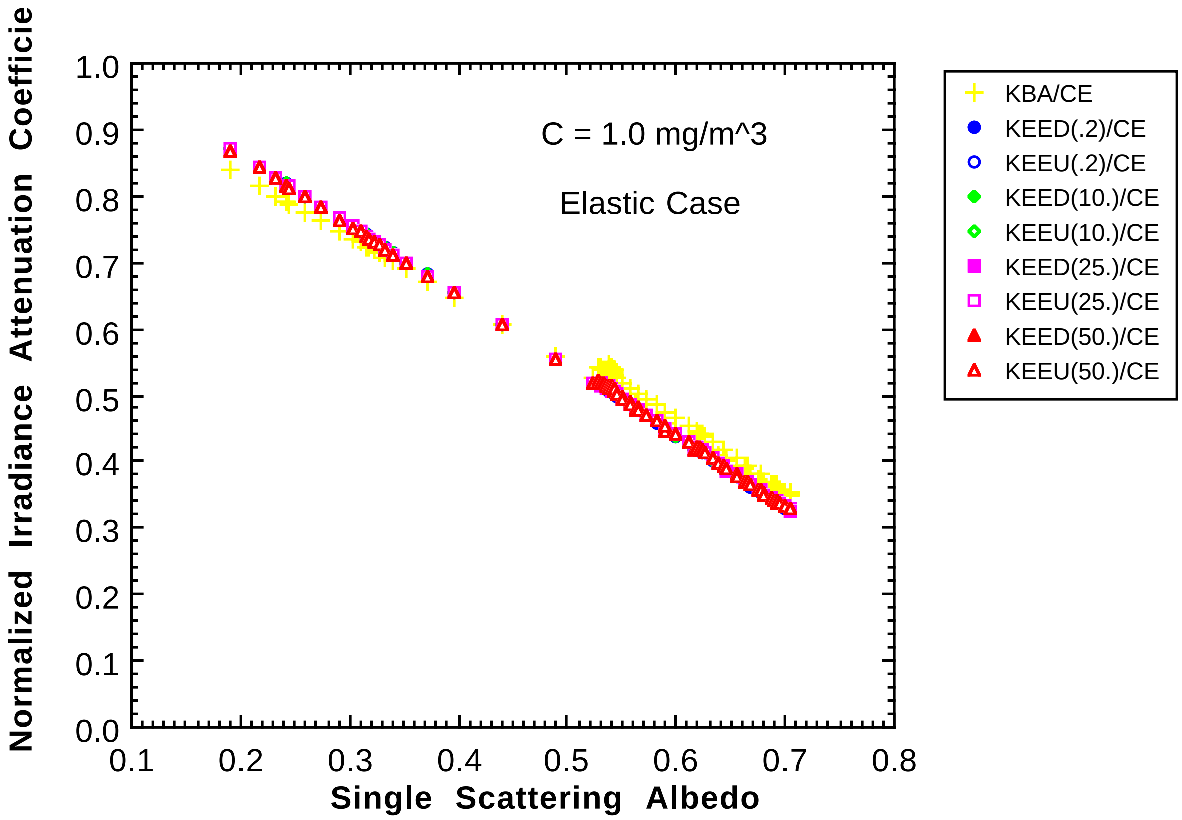

Figure 60. Normalized (by extinction coefficient $c^E=a+b^E$) Irradiance Attenuation Coefficients in high-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are turned off).

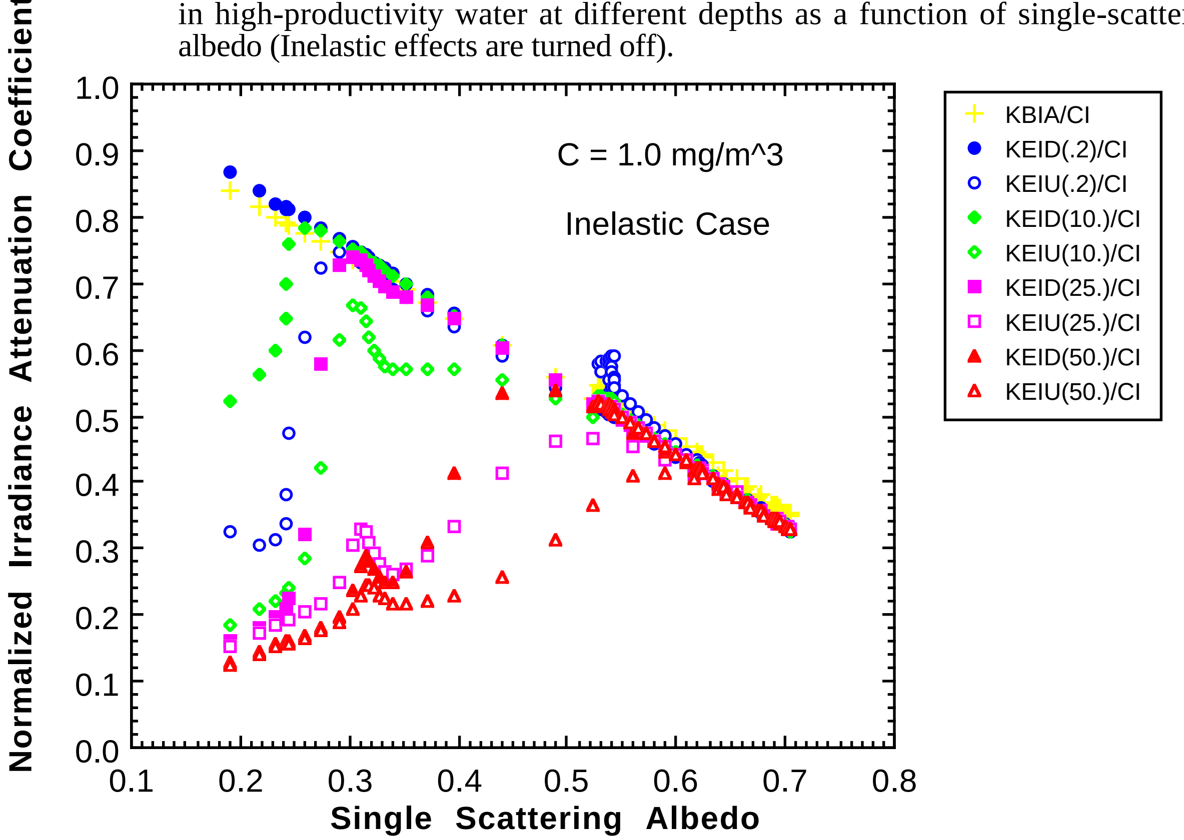

Figure 61. Normalized (by extinction coefficient $c=a+b$) Irradiance Attenuation Coefficients in high-productivity water at different depths as a function of single-scattering albedo (Inelastic effects are taken into account).

8. Conclusion

We have presented a method for calculating irradiances which includes both elastic and inelastic scattering processes (including fluorescence) with an accuracy to no worse than 10% for distilled water and no worse than 5-6% for waters typical of the open ocean [1, 2]. We will present the main observations from the graphs presented from a categorical point of view.

I. Spectral influence of inelastic effects on diffuse reflectance

a) The Raman contribution to the diffuse reflectance is more significant in the blue region of the spectrum; *i.e.*, 400-500 *nm*. The maximum contribution is at 430 *nm* and its contribution decreases with increasing chlorophyll concentration due to the increasing absorption coefficient of the water (see Fig. 10).

b) Blue fluorescence is significant between 400 and 450 *nm* and its value increases with increasing chlorophyll concentration (see Fig. 11).

c) Red fluorescence is significant between 650 and 700 *nm* and its value also increases with increasing chlorophyll concentration (see Fig. 11)

II. Total influence of inelastic effects on diffuse reflectance

a) In the blue region of the spectrum (400–470 *nm*, see Fig. 13)

1) For low productivity (C=0–0.12 mg/m^3) the value of DR increases up to 8%.

2) For medium productivity (C=0.12–0.6 mg/m^3) the value of DR increases between 8–20%.

3) For high productivity waters (C=0.6–3.0 mg/m^3) the value of DR increases between 20-60%.

b) In the green region of the spectrum (470–620 *nm*, see Fig. 14)

1) For low productivity (C=0–0.12 mg/m^3) the value of DR increases between 8–10%.

2) For medium productivity (C=0.12–0.6 mg/m^3) the value of DR increases between 5–8%.

3) For high productivity waters (C=0.6–3.0 mg/m^3) the value of DR increases between 3–5%.

c) In the red region of the spectrum (620–700 *nm*, see Fig. 15)

1) For low productivity (C=0–0.12 mg/m^3) the value of DR increases between 0–200%.

2) For medium productivity (C=0.12–0.6 mg/m^3) the value of DR increases between 200–600%.

3) For high productivity waters (C=0.6–3.0 mg/m^3) the value of DR increases between 600–1000%.

III. Depth dependence of irradiances

a) distilled water

The Raman correction to downward irradiance in blue region of spectrum reaches its maximum at ~ 35 *m* below the surface (see Fig. 16).

b) small productivity water (C=0.05 mg/m^3)

1) The Raman correction to downward irradiance in blue region of spectrum reaches its maximum at ~ 30 *m* below the surface(see Fig. 18).
2) The fluorescent correction to downward irradiance in blue region of spectrum reaches its maximum at ~ 20 *m* below the surface and in red region of spectrum at ~ 3 *m* below the surface (see Fig. 19).

c) medium productivity water (C=0.3 mg/m^3)

1) The Raman correction to downward irradiance in green region of spectrum reaches its maximum at ~15 *m* below the surface (see Fig. 22).
2) The fluorescent correction to downward irradiance in blue region of spectrum reaches its maximum at ~ 10 *m* below the surface and in red region of spectrum at ~ 5 *m* below the surface (see Fig. 23).

d) high productivity water (C=1.0 mg/m^3)

1) The Raman correction to the downward irradiance in the green region of spectrum reaches its maximum at ~ 9 *m* below the surface (see Fig. 26).
2) The fluorescent corrections to the downward irradiances in both the blue and red regions of spectrum becomes maximum at ~3 *m* below the surface (see Fig. 27).

IV. Irradiance attenuation coefficients

The most dramatic effect of inelastic processes is on the irradiance attenuation coefficients in the red part of the spectrum (600-700 *nm*, see Figs. 32, 40, 48, 56). At these wavelengths, natural light penetrates into the water very little; therefore, at depths below 5 *m* almost all light consists of Raman scattered light and in the vicinity of 685 *nm*; chlorophyll fluorescence also contributes significantly. The effective attenuation coefficients at these wavelengths are superpositions of elastic attenuation coefficients and attenuation coefficients averaged over excitation wavelengths, which have significantly lower values since they lie in the transparency window (See Appendix I). Because of the high values of the inherent attenuation coefficients in the red region of the spectrum, its relative weight exponentially decreases, and this leads to a flattening of the spectral dependence of the irradiance attenuation coefficients to values close to inherent values in the transparency window. This occurs at depths approximately equal to or greater than euphotic zone depths (see Fig. 7 and Appendix I.). In other words, in the red region of the spectrum the *vertical* transport of energy over the depth dimension is significantly lower than *horizontal* transport of energy over the spectral dimension from shorter to longer wavelengths.

Therefore at large optical depths (see Appendix I) there is only light which has been transferred inelastically; *i. e.*, both through Raman scattering and fluorescence.

In conclusion we see that neglecting inelastic processes in the ocean can lead to significant errors in predicting values of diffuse reflectances, irradiances, and attenuation coefficients.

This procedure can be extended to handle the more realistic case of a stratified ocean.

9. Acknowledgements

This work was partially supported by the Office of Naval Research under contract number N00014–89–J–1467.

10. Examples of Tables

As an example we give here only a few tables calculated with our code for only one set of optical parameters typical of sea water of medium biological productivity:

$C = 0.3\ mg/m^3$; $Y = 0.3$; $P = 0.203\ g/m^3$; $P_l = 0.194\ g/m^3$; $P_S = 0.009\ g/m^3$; $z_\oplus = 30°$; $\beta_R = 0.0004$; $\eta_C = 0.008$; $\eta_Y = 0.008$.

Table 4.

λ, nm	a, m^{-1}	b^E, m^{-1}	c^E, m^{-1}	b_B, m^{-1}	ω_0	d^E, m	d, m
399.8868	0.0624210	0.0821628	0.1445838	0.0033516	0.5682711	57.40484	57.08662
401.9064	0.0618643	0.0818512	0.1437155	0.0032853	0.5695366	58.00472	57.68698
403.9466	0.0613556	0.0815417	0.1428973	0.0032202	0.5706316	58.57520	58.25837
406.0075	0.0609349	0.0812342	0.1421691	0.0031560	0.5713916	59.08046	58.76535
408.0896	0.0604619	0.0809287	0.1413906	0.0030929	0.5723769	59.63739	59.32351
410.1932	0.0600365	0.0806251	0.1406617	0.0030309	0.5731849	60.16110	59.84890
412.3185	0.0598987	0.0803235	0.1402222	0.0029698	0.5728303	60.43727	60.12941
414.4660	0.0598381	0.0800238	0.1398619	0.0029097	0.5721628	60.64565	60.34281
416.6360	0.0597648	0.0797259	0.1394907	0.0028506	0.5715502	60.86538	60.56742
418.8288	0.0597384	0.0794300	0.1391684	0.0027925	0.5707471	61.04292	60.75021
421.0448	0.0594590	0.0791358	0.1385948	0.0027354	0.5709870	61.44911	61.15938
423.2844	0.0589562	0.0788434	0.1377996	0.0026792	0.5721601	62.06493	61.77622
425.5480	0.0584399	0.0785528	0.1369927	0.0026239	0.5734088	62.70372	62.41590
427.8359	0.0579700	0.0782640	0.1362340	0.0025696	0.5744822	63.30882	63.02230
430.1485	0.0575763	0.0779769	0.1355532	0.0025162	0.5752494	63.84880	63.56427
432.4863	0.0580886	0.0776914	0.1357800	0.0024636	0.5721861	63.50769	63.23307
434.8496	0.0586467	0.0774077	0.1360544	0.0024120	0.5689465	63.12619	62.86152
437.2389	0.0593506	0.0771256	0.1364761	0.0023613	0.5651214	62.61238	62.35845
439.6545	0.0599699	0.0768451	0.1368150	0.0023114	0.5616716	62.18492	61.94067
442.0971	0.0589245	0.0765662	0.1354907	0.0022624	0.5651030	63.31501	63.06782
444.5669	0.0575942	0.0762889	0.1338831	0.0022142	0.5698170	64.76798	64.51546
447.0644	0.0562789	0.0760131	0.1322921	0.0021669	0.5745857	66.27072	66.01265
449.5902	0.0549784	0.0757389	0.1307172	0.0021204	0.5794099	67.82576	67.56192
452.1447	0.0536824	0.0754661	0.1291485	0.0020748	0.5843361	69.44726	69.17732
454.7284	0.0524707	0.0751948	0.1276655	0.0020299	0.5889986	71.04333	70.76770
457.3418	0.0512433	0.0749250	0.1261682	0.0019858	0.5938498	72.73124	72.44939
459.9854	0.0499998	0.0746566	0.1246563	0.0019425	0.5988991	74.51919	74.23056
462.6597	0.0484101	0.0743895	0.1227996	0.0019000	0.6057800	76.87921	76.57948
465.3653	0.0468039	0.0741239	0.1209278	0.0018583	0.6129598	79.41504	79.10301
468.1028	0.0452112	0.0738596	0.1190708	0.0018173	0.6202999	82.09896	81.77366
470.8726	0.0441016	0.0735966	0.1176982	0.0017771	0.6252992	84.14117	83.80805
473.6754	0.0440250	0.0733349	0.1173600	0.0017375	0.6248719	84.48422	84.15717
476.5118	0.0440312	0.0730745	0.1171057	0.0016988	0.6240048	84.68577	84.36583
479.3824	0.0440499	0.0728154	0.1168653	0.0016607	0.6230710	84.86456	84.55179
482.2877	0.0432710	0.0725575	0.1158285	0.0016233	0.6264217	86.43842	86.12237
485.2285	0.0421642	0.0723008	0.1144650	0.0015867	0.6316408	88.67771	88.35365
488.2053	0.0411394	0.0720453	0.1131847	0.0015507	0.6365284	90.87019	90.53876
491.2190	0.0404463	0.0717909	0.1122372	0.0015154	0.6396355	92.48819	92.15394
494.2700	0.0402647	0.0715376	0.1118023	0.0014808	0.6398582	93.09268	92.76324
497.3592	0.0400644	0.0712855	0.1113499	0.0014468	0.6401938	93.74102	93.41609
500.4872	0.0402252	0.0710345	0.1112597	0.0014135	0.6384569	93.63674	93.32155
503.6549	0.0423668	0.0707845	0.1131513	0.0013808	0.6255740	89.59608	89.31610
506.8629	0.0444891	0.0705356	0.1150247	0.0013488	0.6132213	85.93667	85.68676

continued

Table 4 (continuation).

λ, *nm*	*a*, m^{-1}	b^E , m^{-1}	c^E , m^{-1}	b_B , m^{-1}	ω_0	d^E , *m*	*d*, *m*
510.1120	0.0466218	0.0702877	0.1169095	0.0013173	0.6012144	82.55890	82.33511
513.4030	0.0498648	0.0700408	0.1199056	0.0012865	0.5841329	77.83495	77.64211
516.7368	0.0530877	0.0697948	0.1228826	0.0012563	0.5679799	73.66357	73.49611
520.1142	0.0562505	0.0695498	0.1258003	0.0012267	0.5528590	69.99824	69.85163
523.5360	0.0567728	0.0693058	0.1260786	0.0011977	0.5497031	69.54111	69.40060
527.0031	0.0572345	0.0690626	0.1262971	0.0011692	0.5468267	69.15704	69.02212
530.5165	0.0579053	0.0688203	0.1267256	0.0011413	0.5430657	68.54759	68.41893
534.0770	0.0592750	0.0685789	0.1278539	0.0011140	0.5363849	67.20805	67.08808
537.6856	0.0606535	0.0683383	0.1289918	0.0010873	0.5297882	65.91301	65.80108
541.3433	0.0625704	0.0680986	0.1306690	0.0010610	0.5211533	64.15191	64.04912
545.0512	0.0651556	0.0678596	0.1330152	0.0010354	0.5101641	61.88888	61.79617
548.8101	0.0678489	0.0676214	0.1354703	0.0010102	0.4991601	59.69577	59.61218
552.6213	0.0704401	0.0673839	0.1378240	0.0009856	0.4889128	57.73527	57.65952
556.4858	0.0729688	0.0671471	0.1401160	0.0009614	0.4792254	55.94778	55.87887
560.4047	0.0757051	0.0669111	0.1426162	0.0009378	0.4691691	54.13098	54.06849
564.3792	0.0791485	0.0666757	0.1458242	0.0009147	0.4572335	51.99036	51.93454
568.4105	0.0826990	0.0664410	0.1491400	0.0008920	0.4454943	49.95545	49.90557
572.4998	0.0910162	0.0662069	0.1572232	0.0008699	0.4211016	45.69099	45.65064
576.6483	0.1027301	0.0659734	0.1687036	0.0008482	0.3910614	40.78326	40.75220
580.8575	0.1162904	0.0657406	0.1820309	0.0008270	0.3611504	36.28148	36.25773
585.1285	0.1370669	0.0655082	0.2025751	0.0008062	0.3233775	31.03425	31.01748
589.4628	0.1582494	0.0652764	0.2235258	0.0007859	0.2920309	27.05872	27.04641
593.8617	0.1943377	0.0650452	0.2593828	0.0007660	0.2507690	22.21405	22.20604
598.3269	0.2330316	0.0648144	0.2978460	0.0007466	0.2176105	18.64653	18.64108
602.8596	0.2604309	0.0645841	0.3250150	0.0007276	0.1987112	16.75051	16.74626
607.4616	0.2810355	0.0643543	0.3453897	0.0007090	0.1863237	15.56520	15.56165
612.1344	0.2966451	0.0641249	0.3607699	0.0006908	0.1777445	14.77654	14.77345
616.8796	0.3060595	0.0638959	0.3699554	0.0006730	0.1727123	14.34240	14.33959
621.6990	0.3139687	0.0636672	0.3776359	0.0006557	0.1685942	13.99855	13.99596
626.5943	0.3190923	0.0634390	0.3825313	0.0006387	0.1658400	13.78754	13.78512
631.5672	0.3242302	0.0632110	0.3874413	0.0006221	0.1631500	13.58228	13.58001
636.6198	0.3291623	0.0629834	0.3921458	0.0006059	0.1606123	13.39130	13.38916
641.7538	0.3359384	0.0627561	0.3986945	0.0005900	0.1574040	13.13474	13.13276
646.9713	0.3461283	0.0625291	0.4086574	0.0005746	0.1530110	12.76351	12.76171
652.2744	0.3641019	0.0623023	0.4264041	0.0005594	0.1461108	12.15291	12.15133
657.6651	0.3925089	0.0620757	0.4545846	0.0005447	0.1365547	11.29612	11.29480
663.1456	0.4167293	0.0618493	0.4785786	0.0005303	0.1292354	10.65699	10.65586
668.7182	0.4379828	0.0616231	0.4996059	0.0005162	0.1233434	10.15391	10.15292
674.3854	0.4484394	0.0613970	0.5098364	0.0005024	0.1204249	9.92592	9.92501
680.1493	0.4562189	0.0611711	0.5173899	0.0004890	0.1182301	9.76404	9.76319
686.0127	0.4839911	0.0609453	0.5449363	0.0004759	0.1118392	9.21687	9.21614
691.9780	0.5322359	0.0607195	0.5929554	0.0004631	0.1024015	8.39745	8.39687
698.0480	0.6224832	0.0604938	0.6829770	0.0004507	0.0885737	7.19913	7.19872

Table 5.

λ, *nm*	*x*	$\overline{\mu}^E$	R_∞^E, %	R_w^E, %	R_s^E, %	$\alpha^E k_\infty^E$, m^{-1}	$\alpha^E k_0^E$, m^{-1}
399.8868	0.0509572	0.7780955	1.5574780	1.8303263	1.4270708	0.0802228	0.1825775
401.9064	0.0504278	0.7792146	1.5398702	1.8100535	1.4117873	0.0793931	0.1809242
403.9466	0.0498667	0.7804079	1.5212283	1.7885828	1.3955938	0.0786199	0.1794107
406.0075	0.0492434	0.7817420	1.5005503	1.7647579	1.3776167	0.0779475	0.1781526
408.0896	0.0486660	0.7829862	1.4814224	1.7427106	1.3609729	0.0772196	0.1767447
410.1932	0.0480578	0.7843052	1.4613055	1.7195143	1.3434536	0.0765474	0.1754756
412.3185	0.0472386	0.7860965	1.4342530	1.6883062	1.3198696	0.0761976	0.1750390
414.4660	0.0463720	0.7880096	1.4056998	1.6553487	1.2949462	0.0759358	0.1748277
416.6360	0.0455265	0.7898951	1.3778984	1.6232405	1.2706478	0.0756617	0.1745809
418.8288	0.0446586	0.7918501	1.3494258	1.5903384	1.2457301	0.0754416	0.1744719
421.0448	0.0439815	0.7933899	1.3272528	1.5647025	1.2263022	0.0749429	0.1736317
423.2844	0.0434685	0.7945648	1.3104833	1.5453061	1.2115951	0.0741993	0.1721459
425.5480	0.0429703	0.7957130	1.2942188	1.5264873	1.1973194	0.0734434	0.1706220
427.8359	0.0424449	0.7969318	1.2770873	1.5066584	1.1822704	0.0727415	0.1692333
430.1485	0.0418718	0.7982706	1.2584276	1.4850522	1.1658643	0.0721263	0.1680664
432.4863	0.0406865	0.8010708	1.2199313	1.4404500	1.1319684	0.0725137	0.1695269
434.8496	0.0395036	0.8039097	1.1816326	1.3960393	1.0981794	0.0729519	0.1711234
437.2389	0.0382633	0.8069355	1.1416147	1.3495945	1.0627999	0.0735506	0.1731456
439.6545	0.0371128	0.8097896	1.1046182	1.3066188	1.0300226	0.0740561	0.1749257
442.0971	0.0369756	0.8101331	1.1002154	1.3015020	1.0261174	0.0727343	0.1718734
444.5669	0.0370226	0.8100154	1.1017227	1.3032538	1.0274545	0.0711027	0.1679943
447.0644	0.0370761	0.8098814	1.1034411	1.3052508	1.0289786	0.0694904	0.1641588
449.5902	0.0371366	0.8097302	1.1053817	1.3075061	1.0306997	0.0678972	0.1603664
452.1447	0.0372110	0.8095441	1.1077721	1.3102838	1.0328194	0.0663118	0.1565876
454.7284	0.0372457	0.8094576	1.1088848	1.3115768	1.0338061	0.0648221	0.1530540
457.3418	0.0373074	0.8093035	1.1108690	1.3138823	1.0355652	0.0633177	0.1494747
459.9854	0.0373983	0.8090769	1.1137892	1.3172753	1.0381540	0.0617985	0.1458492
462.6597	0.0377667	0.8081617	1.1256316	1.3310328	1.0486478	0.0599015	0.1412189
465.3653	0.0381878	0.8071213	1.1391839	1.3467720	1.0606484	0.0579887	0.1365414
468.1028	0.0386429	0.8060039	1.1538475	1.3637964	1.0736230	0.0560930	0.1319033
470.8726	0.0387342	0.8057805	1.1567932	1.3672156	1.0762281	0.0547316	0.1286679
473.6754	0.0379692	0.8076606	1.1321469	1.3386000	1.0544182	0.0545093	0.1284306
476.5118	0.0371483	0.8097008	1.1057582	1.3079435	1.0310335	0.0543796	0.1284348
479.3824	0.0363313	0.8117554	1.0795583	1.2774878	1.0077818	0.0542650	0.1284768
482.2877	0.0361597	0.8121899	1.0740651	1.2711001	1.0029024	0.0532770	0.1262025
485.2285	0.0362668	0.8119185	1.0774935	1.2750869	1.0059479	0.0519316	0.1229760
488.2053	0.0363253	0.8117704	1.0793677	1.2772662	1.0076125	0.0506786	0.1199878
491.2190	0.0361147	0.8123041	1.0726241	1.2694242	1.0016220	0.0497921	0.1179634
494.2700	0.0354723	0.8139422	1.0520796	1.2455256	0.9833576	0.0494687	0.1174256
497.3592	0.0348541	0.8155334	1.0323497	1.2225636	0.9657965	0.0491266	0.1168342
500.4872	0.0339472	0.8178954	1.0034694	1.1889324	0.9400535	0.0491813	0.1172935
503.6549	0.0315637	0.8242678	0.9279525	1.1008796	0.8725226	0.0513993	0.1235188
506.8629	0.0294252	0.8302058	0.8606887	1.0223069	0.8120925	0.0535880	0.1296978

continued

Table 5 (continuation).

λ, *nm*	*x*	$\bar{\mu}^E$	R^E_∞, %	R^E_w, %	R^E_s, %	$\alpha^E k^E_\infty$, m^{-1}	$\alpha^E k^E_0$, m^{-1}
510.1120	0.0274798	0.8358086	0.7999188	0.9511990	0.7572550	0.0557805	0.1359165
513.4030	0.0251518	0.8427956	0.7277384	0.8665829	0.6918016	0.0591659	0.1453862
516.7368	0.0231182	0.8491818	0.6651940	0.7931188	0.6347854	0.0625163	0.1548123
520.1142	0.0213428	0.8550019	0.6109928	0.7293404	0.5851316	0.0657899	0.1640761
523.5360	0.0206604	0.8573055	0.5902657	0.7049217	0.5660802	0.0662224	0.1656192
527.0031	0.0200201	0.8595038	0.5708672	0.6820533	0.5482171	0.0665901	0.1669866
530.5165	0.0193299	0.8619143	0.5500195	0.6574598	0.5289828	0.0671822	0.1689685
534.0770	0.0184479	0.8650611	0.5234659	0.6261099	0.5044276	0.0685212	0.1730017
537.6856	0.0176105	0.8681215	0.4983538	0.5964351	0.4811450	0.0698675	0.1770644
541.3433	0.0166751	0.8716304	0.4704185	0.5633924	0.4551728	0.0717855	0.1827117
545.0512	0.0156422	0.8756254	0.4397150	0.5270359	0.4265358	0.0744104	0.1903274
548.8101	0.0146708	0.8795090	0.4109799	0.4929713	0.3996437	0.0771441	0.1982685
552.6213	0.0137987	0.8831099	0.3853047	0.4625011	0.3755371	0.0797636	0.2059170
556.4858	0.0130048	0.8864907	0.3620376	0.4348607	0.3536245	0.0823120	0.2133891
560.4047	0.0122363	0.8898661	0.3396096	0.4081911	0.3324390	0.0850747	0.2214785
564.3792	0.0114247	0.8935510	0.3160303	0.3801240	0.3100961	0.0885775	0.2316586
568.4105	0.0106717	0.8970917	0.2942551	0.3541772	0.2893955	0.0921856	0.2421633
572.4998	0.0094672	0.9030324	0.2596338	0.3128672	0.2563417	0.1007896	0.2667419
576.6483	0.0081891	0.9097741	0.2232022	0.2693181	0.2213567	0.1129183	0.3013840
580.8575	0.0070612	0.9161841	0.1913281	0.2311448	0.1905599	0.1269291	0.3415312
585.1285	0.0058476	0.9236931	0.1573459	0.1903660	0.1575099	0.1483901	0.4030949
589.4628	0.0049417	0.9298293	0.1322128	0.1601472	0.1329050	0.1701918	0.4659432
593.8617	0.0039263	0.9374299	0.1042995	0.1265202	0.1053952	0.2073090	0.5731011
598.3269	0.0031936	0.9435542	0.0843471	0.1024366	0.0855966	0.2469721	0.6881369
602.8596	0.0027860	0.9472715	0.0733225	0.0891106	0.0746024	0.2749274	0.7697034
607.4616	0.0025164	0.9498822	0.0660642	0.0803293	0.0673409	0.2958635	0.8311116
612.1344	0.0023234	0.9518400	0.0608814	0.0740549	0.0621436	0.3116543	0.8776867
616.8796	0.0021942	0.9531951	0.0574238	0.0698671	0.0586705	0.3210880	0.9058400
621.6990	0.0020840	0.9543842	0.0544766	0.0662962	0.0557062	0.3289751	0.9295201
626.5943	0.0019976	0.9553383	0.0521707	0.0635015	0.0533844	0.3340097	0.9449122
631.5672	0.0019150	0.9562698	0.0499697	0.0608331	0.0511659	0.3390573	0.9603502
636.6198	0.0018373	0.9571651	0.0479007	0.0583242	0.0490786	0.3438930	0.9751782
641.7538	0.0017533	0.9581542	0.0456678	0.0556159	0.0468238	0.3506100	0.9955023
646.9713	0.0016573	0.9593156	0.0431169	0.0525208	0.0442450	0.3608076	1.0260026
652.2744	0.0015342	0.9608536	0.0398560	0.0485628	0.0409437	0.3789358	1.0797081
657.6651	0.0013858	0.9627928	0.0359340	0.0438000	0.0369659	0.4076774	1.1645358
663.1456	0.0012709	0.9643678	0.0329034	0.0401179	0.0338866	0.4321269	1.2369102
668.7182	0.0011772	0.9657048	0.0304391	0.0371226	0.0313788	0.4535370	1.3004581
674.3854	0.0011192	0.9665595	0.0289155	0.0352702	0.0298265	0.4639542	1.3318123
680.1493	0.0010708	0.9672902	0.0276452	0.0337254	0.0285313	0.4716463	1.3551844
686.0127	0.0009824	0.9686682	0.0253295	0.0309084	0.0261673	0.4996459	1.4382214
691.9780	0.0008695	0.9705229	0.0223772	0.0273153	0.0231484	0.5484011	1.5823944
698.0480	0.0007235	0.9731094	0.0185737	0.0226836	0.0192500	0.6396847	1.8520503

Table 6.

λ, nm	a, m^{-1}	c, m^{-1}	ϖ_0,	$\tilde{x}$	$\bar{\mu}$	R_∞, %	R_s, %
399.8868	0.0628215	0.1449842	0.5667014	0.0506488	0.7787468	1.5472163	1.4181651
401.9064	0.0622561	0.1441074	0.5679878	0.0501263	0.7798549	1.5298506	1.4030853
403.9466	0.0617390	0.1432807	0.5691047	0.0495724	0.7810367	1.5114608	1.3871042
406.0075	0.0613099	0.1425441	0.5698883	0.0489569	0.7823583	1.4910568	1.3693578
408.0896	0.0608287	0.1417574	0.5708957	0.0483867	0.7835908	1.4721802	1.3529260
410.1932	0.0603953	0.1410204	0.5717266	0.0477860	0.7848978	1.4523230	1.3356259
412.3185	0.0602495	0.1405730	0.5714005	0.0469764	0.7866733	1.4256079	1.3123269
414.4660	0.0601812	0.1402050	0.5707628	0.0461199	0.7885699	1.3974028	1.2876978
416.6360	0.0601002	0.1398261	0.5701792	0.0452839	0.7904395	1.3699337	1.2636809
418.8288	0.0600663	0.1394963	0.5694056	0.0444257	0.7923784	1.3417940	1.2390455
421.0448	0.0597794	0.1389152	0.5696696	0.0437560	0.7939054	1.3198786	1.2198364
423.2844	0.0592694	0.1381128	0.5708625	0.0432487	0.7950705	1.3033049	1.2052958
425.5480	0.0587460	0.1372988	0.5721305	0.0427560	0.7962092	1.2872280	1.1911800
427.8359	0.0582691	0.1365330	0.5732238	0.0422363	0.7974181	1.2702908	1.1762965
430.1485	0.0578685	0.1358453	0.5740121	0.0416692	0.7987463	1.2518380	1.1600670
432.4863	0.0583740	0.1360655	0.5709858	0.0404957	0.8015259	1.2137430	1.1265134
434.8496	0.0589255	0.1363332	0.5677831	0.0393240	0.8043446	1.1758308	1.0930548
437.2389	0.0596228	0.1367484	0.5639962	0.0380952	0.8073495	1.1362027	1.0580093
439.6545	0.0602357	0.1370809	0.5605824	0.0369550	0.8101846	1.0995554	1.0255319
442.0971	0.0591841	0.1357503	0.5640225	0.0368194	0.8105249	1.0952045	1.0216716
444.5669	0.0578476	0.1341365	0.5687406	0.0368664	0.8104070	1.0967112	1.0230085
447.0644	0.0565263	0.1325394	0.5735134	0.0369199	0.8102728	1.0984273	1.0245311
449.5902	0.0552197	0.1309586	0.5783420	0.0369802	0.8101214	1.1003640	1.0262492
452.1447	0.0539179	0.1293840	0.5832723	0.0370545	0.8099355	1.1027469	1.0283628
454.7284	0.0527005	0.1278953	0.5879402	0.0370893	0.8098484	1.1038636	1.0293533
457.3418	0.0514674	0.1263924	0.5927965	0.0371510	0.8096941	1.1058443	1.0311099
459.9854	0.0502184	0.1248750	0.5978504	0.0372415	0.8094678	1.1087531	1.0336893
462.6597	0.0486233	0.1230129	0.6047298	0.0376073	0.8085571	1.1205056	1.0441064
465.3653	0.0470119	0.1211358	0.6119075	0.0380253	0.8075220	1.1339525	1.0560170
468.1028	0.0454139	0.1192735	0.6192455	0.0384770	0.8064104	1.1485003	1.0688928
470.8726	0.0442993	0.1178959	0.6242510	0.0385681	0.8061871	1.1514359	1.0714898
473.6754	0.0442177	0.1175526	0.6238479	0.0378101	0.8080543	1.1270260	1.0498830
476.5118	0.0442189	0.1172935	0.6230061	0.0369964	0.8100809	1.1008831	1.0267097
479.3824	0.0442328	0.1170482	0.6220973	0.0361865	0.8121221	1.0749212	1.0036629
482.2877	0.0434492	0.1160067	0.6254594	0.0360168	0.8125528	1.0694894	0.9988366
485.2285	0.0423378	0.1146386	0.6306844	0.0361235	0.8122818	1.0729049	1.0018715
488.2053	0.0413085	0.1133537	0.6355790	0.0361820	0.8121333	1.0747795	1.0035370
491.2190	0.0406109	0.1124018	0.6386987	0.0359736	0.8126626	1.0681078	0.9976088
494.2700	0.0404250	0.1119626	0.6389423	0.0353366	0.8142902	1.0477464	0.9795024
497.3592	0.0402204	0.1115059	0.6392981	0.0347236	0.8158713	1.0281890	0.9620905
500.4872	0.0403770	0.1114115	0.6375867	0.0338239	0.8182192	0.9995478	0.9365545
503.6549	0.0425146	0.1132991	0.6247581	0.0314574	0.8245577	0.9245996	0.8695167
506.8629	0.0446328	0.1151684	0.6124559	0.0293332	0.8304662	0.8578058	0.8094964

continued

Table 6 (continuation).

λ, *nm*	*a, m*$^{-1}$	*c, m*$^{-1}$	ϖ_0,	$\tilde{x}$	$\bar{\mu}$	R_∞ , %	R_s, %
510.1120	0.0467617	0.1170493	0.6004961	0.0273999	0.8360432	0.7974308	0.7550047
513.4030	0.0500008	0.1200416	0.5834710	0.0250851	0.8430006	0.7256794	0.6899292
516.7368	0.0532200	0.1230148	0.5673693	0.0230621	0.8493621	0.6634748	0.6332139
520.1142	0.0563791	0.1259289	0.5522945	0.0212951	0.8551615	0.6095438	0.5838008
523.5360	0.0568978	0.1262036	0.5491586	0.0206160	0.8574570	0.5889175	0.5648397
527.0031	0.0573559	0.1264186	0.5463012	0.0199785	0.8596477	0.5696102	0.5470585
530.5165	0.0580233	0.1268437	0.5425603	0.0192914	0.8620503	0.5488564	0.5279086
534.0770	0.0593897	0.1279686	0.5359042	0.0184129	0.8651874	0.5224154	0.5034549
537.6856	0.0607649	0.1291032	0.5293311	0.0175787	0.8682389	0.4974047	0.4802638
541.3433	0.0626786	0.1307771	0.5207222	0.0166468	0.8717381	0.4695752	0.4543877
545.0512	0.0652607	0.1331202	0.5097616	0.0156175	0.8757229	0.4389803	0.4258493
548.8101	0.0679509	0.1355722	0.4987848	0.0146491	0.8795972	0.4103399	0.3990438
552.6213	0.0705390	0.1379229	0.4885620	0.0137796	0.8831900	0.3847441	0.3750098
556.4858	0.0730649	0.1402120	0.4788973	0.0129880	0.8865637	0.3615443	0.3531592
560.4047	0.0757982	0.1427093	0.4688629	0.0122215	0.8899324	0.3391771	0.3320299
564.3792	0.0792388	0.1459146	0.4569504	0.0114119	0.8936106	0.3156571	0.3097419
568.4105	0.0827866	0.1492276	0.4452327	0.0106606	0.8971452	0.2939328	0.2890886
572.4998	0.0911012	0.1573081	0.4208742	0.0094584	0.9030769	0.2593835	0.2561021
576.6483	0.1028124	0.1687859	0.3908706	0.0081826	0.9098097	0.2230177	0.2211789
580.8575	0.1163702	0.1821107	0.3609922	0.0070564	0.9162125	0.1911929	0.1904289
585.1285	0.1371442	0.2026524	0.3232541	0.0058443	0.9237144	0.1572546	0.1574208
589.4628	0.1583242	0.2236007	0.2919331	0.0049394	0.9298458	0.1321486	0.1328420
593.8617	0.1944102	0.2594553	0.2506990	0.0039248	0.9374415	0.1042596	0.1053557
598.3269	0.2331018	0.2979162	0.2175592	0.0031927	0.9435627	0.0843211	0.0855707
602.8596	0.2604988	0.3250830	0.1986697	0.0027853	0.9472784	0.0733029	0.0745829
607.4616	0.2811012	0.3454555	0.1862882	0.0025159	0.9498881	0.0660485	0.0673251
612.1344	0.2967087	0.3608335	0.1777132	0.0023229	0.9518451	0.0608681	0.0621303
616.8796	0.3061210	0.3700169	0.1726836	0.0021938	0.9531997	0.0574121	0.0586587
621.6990	0.3140281	0.3776954	0.1685677	0.0020836	0.9543886	0.0544661	0.0556956
626.5943	0.3191498	0.3825887	0.1658150	0.0019973	0.9553423	0.0521611	0.0533747
631.5672	0.3242858	0.3874968	0.1631266	0.0019147	0.9562735	0.0499609	0.0511571
636.6198	0.3292160	0.3921995	0.1605903	0.0018370	0.9571685	0.0478927	0.0490706
641.7538	0.3359903	0.3987464	0.1573835	0.0017531	0.9581574	0.0456607	0.0468166
646.9713	0.3461784	0.4087074	0.1529922	0.0016570	0.9593185	0.0431105	0.0442385
652.2744	0.3641502	0.4264524	0.1460943	0.0015340	0.9608562	0.0398506	0.0409382
657.6651	0.3925555	0.4546312	0.1365407	0.0013856	0.9627950	0.0359296	0.0369615
663.1456	0.4167743	0.4786236	0.1292232	0.0012707	0.9643697	0.0328997	0.0338829
668.7182	0.4380262	0.4996493	0.1233327	0.0011771	0.9657065	0.0304360	0.0313757
674.3854	0.4484812	0.5098783	0.1204151	0.0011191	0.9665611	0.0289127	0.0298237
680.1493	0.4562592	0.5174303	0.1182209	0.0010707	0.9672916	0.0276427	0.0285287
686.0127	0.4840299	0.5449752	0.1118313	0.0009823	0.9686694	0.0253274	0.0261652
691.9780	0.5322733	0.5929928	0.1023950	0.0008694	0.9705240	0.0223756	0.0231468
698.0480	0.6225192	0.6830130	0.0885691	0.0007235	0.9731102	0.0185726	0.0192489

Table 7.

λ, *nm*	R_w, %	α, m^{-1}	δR^R, %	δR^{CY}, %	R, %	αk_∞, m^{-1}	αk_0, m^{-1}
399.8868	1.8185123	0.0695246	0.0555263	0.0288482	1.5025396	0.0806700	0.1837341
401.9064	1.7985145	0.0688268	0.0543558	0.0355379	1.4929790	0.0798304	0.1820561
403.9466	1.7773301	0.0681793	0.0534942	0.0430302	1.4836286	0.0790474	0.1805184
406.0075	1.7538165	0.0676220	0.0526933	0.0510420	1.4730931	0.0783655	0.1792364
408.0896	1.7320548	0.0670146	0.0524790	0.0593906	1.4647956	0.0776282	0.1778050
410.1932	1.7091538	0.0664571	0.0531256	0.0678370	1.4565885	0.0769467	0.1765128
412.3185	1.6783296	0.0661892	0.0552159	0.0768566	1.4443994	0.0765877	0.1760537
414.4660	1.6457682	0.0660007	0.0579257	0.0853079	1.4309314	0.0763169	0.1758201
416.6360	1.6140387	0.0658015	0.0610024	0.0928808	1.4175640	0.0760339	0.1755515
418.8288	1.5815160	0.0656514	0.0637189	0.0991150	1.4018793	0.0758051	0.1754211
421.0448	1.5561740	0.0652502	0.0665384	0.1049333	1.3913081	0.0752979	0.1745597
423.2844	1.5370011	0.0646278	0.0694871	0.1103552	1.3851382	0.0745461	0.1730531
425.5480	1.5183967	0.0639938	0.0718322	0.1138713	1.3768835	0.0737821	0.1715087
427.8359	1.4987897	0.0634083	0.0737644	0.1152541	1.3653150	0.0730722	0.1700999
430.1485	1.4774202	0.0629008	0.0749783	0.1138614	1.3489067	0.0724491	0.1689132
432.4863	1.4332766	0.0633013	0.0705338	0.1027993	1.2998465	0.0728286	0.1703545
434.8496	1.3893083	0.0637496	0.0661283	0.0911140	1.2502972	0.0732591	0.1719321
437.2389	1.3433101	0.0643454	0.0617559	0.0791944	1.1989596	0.0738501	0.1739358
439.6545	1.3007349	0.0648586	0.0576216	0.0676444	1.1507979	0.0743482	0.1756977
442.0971	1.2956778	0.0637089	0.0553783	0.0583338	1.1353836	0.0730194	0.1726272
444.5669	1.2974291	0.0622762	0.0533289	0.0495874	1.1259248	0.0713810	0.1687301
447.0644	1.2994238	0.0608602	0.0511564	0.0413329	1.1170204	0.0697620	0.1648770
449.5902	1.3016747	0.0594607	0.0489645	0.0337917	1.1090054	0.0681623	0.1610674
452.1447	1.3044440	0.0580675	0.0481419	0.0276667	1.1041714	0.0665706	0.1572715
454.7284	1.3057418	0.0567603	0.0479975	0.0222826	1.0996334	0.0650746	0.1537213
457.3418	1.3080436	0.0554391	0.0484790	0.0176025	1.0971914	0.0635640	0.1501256
459.9854	1.3114238	0.0541035	0.0494803	0.0136365	1.0968061	0.0620388	0.1464841
462.6597	1.3250783	0.0524234	0.0514511	0.0104890	1.1060465	0.0601359	0.1418380
465.3653	1.3406969	0.0507285	0.0535939	0.0079148	1.1175257	0.0582175	0.1371450
468.1028	1.3575889	0.0490486	0.0557809	0.0058549	1.1305287	0.0563161	0.1324917
470.8726	1.3609968	0.0478534	0.0574414	0.0042120	1.1331432	0.0549491	0.1292414
473.6754	1.3326524	0.0476928	0.0576805	0.0029188	1.1104822	0.0547212	0.1289898
476.5118	1.3022780	0.0476165	0.0575482	0.0019815	1.0862394	0.0545858	0.1289799
479.3824	1.2720957	0.0475543	0.0570720	0.0013191	1.0620540	0.0544658	0.1290081
482.2877	1.2657785	0.0466960	0.0579411	0.0008822	1.0576600	0.0534725	0.1267202
485.2285	1.2697508	0.0455112	0.0592109	0.0005828	1.0616653	0.0521221	0.1234803
488.2053	1.2719309	0.0444099	0.0603677	0.0003790	1.0642837	0.0508642	0.1204789
491.2190	1.2641715	0.0436418	0.0606303	0.0002387	1.0584778	0.0499727	0.1184417
494.2700	1.2404834	0.0433866	0.0595439	0.0001437	1.0391899	0.0496444	0.1178913
497.3592	1.2177199	0.0431141	0.0584609	0.0000850	1.0206364	0.0492975	0.1172877
500.4872	1.1843639	0.0432040	0.0570757	0.0000505	0.9936806	0.0493474	0.1177350
503.6549	1.0969662	0.0452762	0.0541432	0.0000277	0.9236875	0.0515604	0.1239490
506.8629	1.0189362	0.0473304	0.0513310	0.0000155	0.8608429	0.0537443	0.1301167

continued

Table 7 (continuation).

λ, *nm*	R_w, %	α, m^{-1}	δR^R, %	δR^{CY}, %	R, %	αk_∞, m^{-1}	αk_0, m^{-1}
510.1120	0.9482851	0.0493964	0.0485679	0.0000076	0.8035802	0.0559321	0.1363244
513.4030	0.8641666	0.0525739	0.0452452	0.0000037	0.7351781	0.0593129	0.1457834
516.7368	0.7910974	0.0557327	0.0421247	0.0000018	0.6753404	0.0626588	0.1551990
520.1142	0.7276338	0.0588325	0.0392685	0.0000018	0.6230711	0.0659280	0.1644525
523.5360	0.7033328	0.0592932	0.0373818	0.0000000	0.6022216	0.0663564	0.1659852
527.0031	0.6805710	0.0596944	0.0356759	0.0000000	0.5827343	0.0667203	0.1673424
530.5165	0.6560873	0.0603061	0.0340305	0.0000000	0.5619391	0.0673085	0.1693144
534.0770	0.6248691	0.0616178	0.0326612	0.0000000	0.5361161	0.0686437	0.1733379
537.6856	0.5953130	0.0629394	0.0312297	0.0000000	0.5114935	0.0699863	0.1773912
541.3433	0.5623945	0.0648007	0.0294411	0.0000000	0.4838288	0.0719007	0.1830292
545.0512	0.5261654	0.0673314	0.0272842	0.0000000	0.4531335	0.0745220	0.1906359
548.8101	0.4922122	0.0699713	0.0252228	0.0000000	0.4242666	0.0772523	0.1985682
552.6213	0.4618354	0.0725102	0.0236295	0.0000000	0.3986394	0.0798684	0.2062080
556.4858	0.4342745	0.0749878	0.0223870	0.0000000	0.3755462	0.0824135	0.2136716
560.4047	0.4076766	0.0776739	0.0213243	0.0000000	0.3533542	0.0851730	0.2217528
564.3792	0.3796795	0.0810683	0.0202307	0.0000000	0.3299725	0.0886727	0.2319248
568.4105	0.3537929	0.0845707	0.0192155	0.0000000	0.3083042	0.0922778	0.2424216
572.4998	0.3125683	0.0928410	0.0173134	0.0000000	0.2734154	0.1008786	0.2669925
576.6483	0.2690973	0.1045089	0.0151367	0.0000000	0.2363156	0.1130043	0.3016273
580.8575	0.2309827	0.1180242	0.0132471	0.0000000	0.2036760	0.1270122	0.3417672
585.1285	0.1902564	0.1387566	0.0114452	0.0000000	0.1688661	0.1484703	0.4033239
589.4628	0.1600699	0.1598960	0.0100336	0.0000000	0.1428755	0.1702693	0.4661651
593.8617	0.1264721	0.1959422	0.0080117	0.0000000	0.1133674	0.2073838	0.5733163
598.3269	0.1024052	0.2345949	0.0064609	0.0000000	0.0920316	0.2470443	0.6883454
602.8596	0.0890869	0.2619540	0.0055617	0.0000000	0.0801446	0.2749971	0.7699054
607.4616	0.0803102	0.2825192	0.0049491	0.0000000	0.0722742	0.2959309	0.8313071
612.1344	0.0740387	0.2980903	0.0045094	0.0000000	0.0666396	0.3117195	0.8778759
616.8796	0.0698529	0.3074671	0.0042115	0.0000000	0.0628703	0.3211510	0.9060230
621.6990	0.0662834	0.3153395	0.0039565	0.0000000	0.0596520	0.3290359	0.9296972
626.5943	0.0634899	0.3204272	0.0037622	0.0000000	0.0571369	0.3340685	0.9450834
631.5672	0.0608225	0.3255300	0.0035313	0.0000000	0.0546885	0.3391141	0.9605157
636.6198	0.0583146	0.3304278	0.0032461	0.0000005	0.0523173	0.3439478	0.9753381
641.7538	0.0556072	0.3371704	0.0030014	0.0000057	0.0498236	0.3506629	0.9956568
646.9713	0.0525131	0.3473275	0.0028141	0.0000608	0.0471135	0.3608587	1.0261517
652.2744	0.0485563	0.3652691	0.0025808	0.0004439	0.0439629	0.3789851	1.0798522
657.6651	0.0437947	0.3936449	0.0022926	0.0022874	0.0415415	0.4077249	1.1646748
663.1456	0.0401135	0.4178348	0.0020508	0.0086916	0.0446252	0.4321727	1.2370444
668.7182	0.0371189	0.4390586	0.0018374	0.0242755	0.0574885	0.4535811	1.3005876
674.3854	0.0352668	0.4494861	0.0017367	0.0526136	0.0841740	0.4639968	1.3319371
680.1493	0.0337224	0.4572373	0.0016721	0.0850432	0.1152440	0.4716873	1.3553048
686.0127	0.0309059	0.4849818	0.0016163	0.1003538	0.1281353	0.4996854	1.4383374
691.9780	0.0273134	0.5331996	0.0014793	0.0817755	0.1064016	0.5484391	1.5825062
698.0480	0.0226822	0.6234206	0.0012088	0.0433545	0.0638123	0.6397212	1.8521579

Depth Dependent Data

Table 8.

Wavelength, λ = 425.55 nm.

z, m	E_d^q	E_d^s	E_d^E	E_u^s	$\alpha^E k_d^E$, *m-1*	$\alpha^E k_u^E$, *m-1*	$R_s^E(z)$, %
0.01	83.189922	0.002426	83.192348	0.996080	0.065730	0.065494	1.197322
0.10	82.677551	0.024103	82.701654	0.990226	0.065732	0.065496	1.197348
0.20	82.111950	0.047865	82.159815	0.983762	0.065734	0.065499	1.197376
0.30	81.550218	0.071289	81.621508	0.977339	0.065736	0.065501	1.197404
0.50	80.438258	0.117139	80.555397	0.964619	0.065741	0.065506	1.197460
0.70	79.341459	0.161682	79.503140	0.952063	0.065745	0.065511	1.197516
1.00	77.724238	0.226103	77.950341	0.933534	0.065752	0.065518	1.197600
2.00	72.567824	0.421195	72.989019	0.874320	0.065774	0.065543	1.197879
3.00	67.753500	0.588468	68.341967	0.818842	0.065796	0.065567	1.198154
5.00	59.061843	0.850882	59.912725	0.718171	0.065839	0.065615	1.198696
7.00	51.485182	1.033473	52.518655	0.629817	0.065881	0.065662	1.199226
10.00	41.903003	1.193053	43.096056	0.517153	0.065943	0.065731	1.200002
15.00	29.729205	1.254677	30.983882	0.372192	0.066042	0.065840	1.201243
20.00	21.092179	1.172931	22.265110	0.267721	0.066136	0.065944	1.202424
25.00	14.964410	1.028028	15.992438	0.192477	0.066225	0.066043	1.203548
30.00	10.616901	0.865027	11.481928	0.138314	0.066311	0.066137	1.204620
35.00	7.532444	0.707686	8.240130	0.099346	0.066392	0.066226	1.205641
40.00	5.344094	0.567176	5.911269	0.071326	0.066470	0.066312	1.206617
50.00	2.689988	0.348706	3.038695	0.036721	0.066615	0.066471	1.208439
60.00	1.354025	0.205853	1.559878	0.018876	0.066747	0.066616	1.210107
70.00	0.681558	0.118169	0.799727	0.009690	0.066869	0.066748	1.211637
80.00	0.343067	0.066462	0.409530	0.004968	0.066981	0.066870	1.213045
90.00	0.172685	0.036804	0.209489	0.002544	0.067085	0.066982	1.214343
100.00	0.086922	0.020132	0.107055	0.001301	0.067180	0.067085	1.215542
110.00	0.043753	0.010905	0.054658	0.000665	0.067268	0.067181	1.216652
120.00	0.022023	0.005859	0.027882	0.000340	0.067350	0.067269	1.217681
130.00	0.011086	0.003127	0.014212	0.000173	0.067427	0.067351	1.218637
140.00	0.005580	0.001659	0.007239	0.000088	0.067497	0.067427	1.219526
150.00	0.002809	0.000876	0.003685	0.000045	0.067563	0.067498	1.220354
175.00	0.000505	0.000174	0.000679	0.000008	0.067709	0.067654	1.222190
200.00	0.000091	0.000034	0.000125	0.000002	0.067833	0.067786	1.223745

Table 9.

Wavelength, λ = 425.55 nm.

z, m	E_d^R	E_u^R	E_d^{CY}	E_u^{CY}	αk_d, m^{-1}	αk_u, m^{-1}	$R(z)$, %
0.01	0.000148	0.059757	0.000298	0.094665	0.065523	0.071843	1.191183
0.10	0.001475	0.059384	0.002956	0.093512	0.065528	0.071796	1.191208
0.20	0.002929	0.058972	0.005847	0.092249	0.065534	0.071744	1.191236
0.30	0.004361	0.058563	0.008674	0.091005	0.065539	0.071693	1.191264
0.50	0.007162	0.057755	0.014144	0.088572	0.065550	0.071592	1.191320
0.70	0.009880	0.056958	0.019372	0.086211	0.065561	0.071493	1.191375
1.00	0.013806	0.055783	0.026781	0.082799	0.065578	0.071349	1.191459
2.00	0.025655	0.052045	0.048029	0.072463	0.065630	0.070902	1.191734
3.00	0.035756	0.048563	0.064637	0.063539	0.065679	0.070501	1.192007
5.00	0.051453	0.042295	0.086863	0.049127	0.065769	0.069819	1.192544
7.00	0.062203	0.036849	0.098286	0.038255	0.065851	0.069270	1.193070
10.00	0.071323	0.029982	0.102490	0.026617	0.065961	0.068642	1.193838
15.00	0.074208	0.021282	0.092111	0.014981	0.066120	0.067959	1.195069
20.00	0.068676	0.015116	0.074673	0.008704	0.066256	0.067565	1.196239
25.00	0.059617	0.010738	0.057493	0.005190	0.066376	0.067340	1.197353
30.00	0.049702	0.007627	0.042966	0.003161	0.066485	0.067219	1.198414
35.00	0.040297	0.005416	0.031510	0.001959	0.066585	0.067160	1.199427
40.00	0.032010	0.003844	0.022814	0.001231	0.066678	0.067141	1.200393
50.00	0.019333	0.001933	0.011665	0.000502	0.066846	0.067168	1.202199
60.00	0.011208	0.000970	0.005839	0.000211	0.066997	0.067238	1.203851
70.00	0.006315	0.000485	0.002885	0.000091	0.067132	0.067325	1.205367
80.00	0.003484	0.000242	0.001413	0.000040	0.067256	0.067416	1.206761
90.00	0.001891	0.000121	0.000687	0.000018	0.067368	0.067507	1.208047
100.00	0.001013	0.000060	0.000333	0.000008	0.067472	0.067595	1.209234
110.00	0.000537	0.000030	0.000161	0.000004	0.067567	0.067678	1.210334
120.00	0.000282	0.000015	0.000077	0.000002	0.067655	0.067757	1.211353
130.00	0.000147	0.000007	0.000037	0.000001	0.067736	0.067830	1.212299
140.00	0.000076	0.000004	0.000018	0.000000	0.067811	0.067900	1.213179
150.00	0.000039	0.000002	0.000009	0.000000	0.067881	0.067964	1.213999
175.00	0.000007	0.000000	0.000001	0.000000	0.068036	0.068109	1.215818
200.00	0.000001	0.000000	0.000000	0.000000	0.068165	0.068230	1.217356

Table 10.

Wavelength, λ = 497.36 nm.

z, m	E_d^q	E_d^s	E_d^E	E_u^s	$\alpha^E k_d^E$, m-1	$\alpha^E k_u^E$, m-1	$R_s^E(z)$, %
0.01	101.304963	0.001620	101.306582	0.978416	0.044703	0.044593	0.965798
0.10	100.883684	0.016127	100.899811	0.974498	0.044704	0.044594	0.965807
0.20	100.417651	0.032101	100.449751	0.970162	0.044705	0.044595	0.965818
0.30	99.953771	0.047922	100.001692	0.965845	0.044705	0.044595	0.965828
0.50	99.032429	0.079111	99.111540	0.957268	0.044707	0.044597	0.965850
0.70	98.119580	0.109704	98.229283	0.948768	0.044708	0.044599	0.965871
1.00	96.766059	0.154492	96.920551	0.936158	0.044710	0.044601	0.965903
2.00	92.387730	0.294588	92.682318	0.895319	0.044717	0.044609	0.966008
3.00	88.207505	0.421293	88.628799	0.856254	0.044724	0.044616	0.966113
5.00	80.405921	0.638251	81.044171	0.783147	0.044738	0.044631	0.966321
7.00	73.294353	0.812229	74.106583	0.716259	0.044752	0.044646	0.966526
10.00	63.788773	1.005592	64.794365	0.626451	0.044772	0.044668	0.966829
15.00	50.605820	1.188285	51.794105	0.501016	0.044805	0.044704	0.967323
20.00	40.147332	1.248172	41.395503	0.400627	0.044837	0.044739	0.967803
25.00	31.850254	1.229154	33.079409	0.320298	0.044868	0.044773	0.968269
30.00	25.267899	1.162029	26.429927	0.256033	0.044898	0.044806	0.968723
35.00	20.045890	1.068071	21.113961	0.204629	0.044927	0.044837	0.969163
40.00	15.903092	0.961692	16.864783	0.163520	0.044956	0.044868	0.969592
50.00	10.009076	0.746199	10.755276	0.104371	0.045010	0.044928	0.970414
60.00	6.299505	0.555871	6.855376	0.066579	0.045062	0.044984	0.971193
70.00	3.964778	0.402613	4.367391	0.042448	0.045111	0.045037	0.971932
80.00	2.495349	0.285677	2.781026	0.027049	0.045158	0.045087	0.972633
90.00	1.570521	0.199550	1.770071	0.017228	0.045202	0.045135	0.973299
100.00	0.988453	0.137677	1.126131	0.010968	0.045244	0.045181	0.973933
110.00	0.622112	0.094045	0.716157	0.006979	0.045284	0.045224	0.974536
120.00	0.391544	0.063714	0.455259	0.004439	0.045322	0.045265	0.975111
130.00	0.246430	0.042868	0.289298	0.002823	0.045359	0.045304	0.975658
140.00	0.155098	0.028674	0.183772	0.001794	0.045393	0.045341	0.976181
150.00	0.097615	0.019083	0.116699	0.001140	0.045427	0.045377	0.976680
175.00	0.030676	0.006772	0.037448	0.000366	0.045503	0.045459	0.977832
200.00	0.009640	0.002355	0.011995	0.000117	0.045572	0.045532	0.978863

Table 11.

Wavelength, λ = 497.36 nm.

z, m	E_d^R	E_u^R	E_d^{CY}	E_u^{CY}	αk_d, m^{-1}	αk_u, m^{-1}	$R(z)$, %
0.01	0.000102	0.059221	0.000000	0.000086	0.044771	0.045128	0.962092
0.10	0.001016	0.058950	0.000002	0.000085	0.044771	0.045129	0.962101
0.20	0.002023	0.058650	0.000004	0.000084	0.044772	0.045129	0.962112
0.30	0.003018	0.058351	0.000006	0.000083	0.044773	0.045129	0.962122
0.50	0.004980	0.057759	0.000011	0.000081	0.044775	0.045130	0.962143
0.70	0.006900	0.057172	0.000015	0.000078	0.044776	0.045131	0.962164
1.00	0.009708	0.056304	0.000020	0.000075	0.044779	0.045132	0.962196
2.00	0.018448	0.053505	0.000037	0.000066	0.044787	0.045135	0.962301
3.00	0.026293	0.050845	0.000050	0.000058	0.044795	0.045138	0.962405
5.00	0.039562	0.045919	0.000069	0.000045	0.044812	0.045145	0.962611
7.00	0.050002	0.041474	0.000080	0.000035	0.044828	0.045153	0.962815
10.00	0.061269	0.035604	0.000087	0.000024	0.044851	0.045164	0.963116
15.00	0.071154	0.027617	0.000084	0.000014	0.044890	0.045184	0.963607
20.00	0.073444	0.021430	0.000074	0.000008	0.044927	0.045204	0.964083
25.00	0.071065	0.016634	0.000062	0.000005	0.044962	0.045225	0.964546
30.00	0.066007	0.012915	0.000051	0.000003	0.044997	0.045245	0.964996
35.00	0.059604	0.010030	0.000041	0.000002	0.045030	0.045266	0.965434
40.00	0.052721	0.007791	0.000033	0.000001	0.045063	0.045286	0.965859
50.00	0.039472	0.004703	0.000020	0.000001	0.045125	0.045327	0.966676
60.00	0.028368	0.002841	0.000013	0.000000	0.045183	0.045367	0.967449
70.00	0.019820	0.001717	0.000008	0.000000	0.045238	0.045405	0.968183
80.00	0.013564	0.001038	0.000005	0.000000	0.045290	0.045443	0.968879
90.00	0.009137	0.000627	0.000003	0.000000	0.045339	0.045479	0.969541
100.00	0.006078	0.000379	0.000002	0.000000	0.045385	0.045514	0.970170
110.00	0.004003	0.000229	0.000001	0.000000	0.045429	0.045547	0.970769
120.00	0.002614	0.000139	0.000001	0.000000	0.045470	0.045579	0.971339
130.00	0.001695	0.000084	0.000000	0.000000	0.045510	0.045610	0.971883
140.00	0.001093	0.000051	0.000000	0.000000	0.045547	0.045639	0.972402
150.00	0.000701	0.000031	0.000000	0.000000	0.045583	0.045667	0.972897
175.00	0.000226	0.000009	0.000000	0.000000	0.045664	0.045733	0.974040
200.00	0.000072	0.000002	0.000000	0.000000	0.045737	0.045793	0.975064

Table 12.

Wavelength, λ = 691.98 nm.

z, m	E_d^q	E_d^s	E_d^E	E_u^s	$\alpha^E k_d^E$, m^{-1}	$\alpha^E k_u^E$, m^{-1}	$R_s^E(z)$, %
0.01	64.660457	0.000323	64.660780	0.014968	0.574166	0.574183	0.023148
0.10	61.401231	0.003071	61.404303	0.014214	0.574165	0.574182	0.023148
0.20	57.972182	0.005807	57.977989	0.013421	0.574164	0.574181	0.023148
0.30	54.734634	0.008235	54.742869	0.012672	0.574163	0.574179	0.023148
0.50	48.791857	0.012267	48.804124	0.011297	0.574160	0.574177	0.023148
0.70	43.494314	0.015349	43.509663	0.010072	0.574158	0.574174	0.023148
1.00	36.606667	0.018528	36.625195	0.008478	0.574154	0.574171	0.023148
2.00	20.605628	0.021137	20.626765	0.004775	0.574140	0.574158	0.023148
3.00	11.598759	0.018085	11.616844	0.002689	0.574126	0.574144	0.023147
5.00	3.675047	0.009809	3.684856	0.000853	0.574098	0.574117	0.023146
7.00	1.164432	0.004470	1.168902	0.000271	0.574068	0.574088	0.023145
10.00	0.207678	0.001186	0.208865	0.000048	0.574020	0.574042	0.023144
15.00	0.011736	0.000108	0.011844	0.000003	0.573932	0.573956	0.023141
20.00	0.000663	0.000009	0.000672	0.000000	0.573832	0.573860	0.023138
25.00	0.000037	0.000001	0.000038	0.000000	0.573720	0.573751	0.023135
30.00	0.000002	0.000000	0.000002	0.000000	0.573592	0.573627	0.023131
35.00	0.000000	0.000000	0.000000	0.000000	0.573448	0.573488	0.023127
40.00	0.000000	0.000000	0.000000	0.000000	0.573287	0.573331	0.023122
50.00	0.000000	0.000000	0.000000	0.000000	0.572900	0.572956	0.023111
60.00	0.000000	0.000000	0.000000	0.000000	0.572415	0.572485	0.023096
70.00	0.000000	0.000000	0.000000	0.000000	0.571812	0.571899	0.023078
80.00	0.000000	0.000000	0.000000	0.000000	0.571072	0.571178	0.023056
90.00	0.000000	0.000000	0.000000	0.000000	0.570177	0.570305	0.023029
100.00	0.000000	0.000000	0.000000	0.000000	0.569114	0.569264	0.022997
110.00	0.000000	0.000000	0.000000	0.000000	0.567878	0.568050	0.022960
120.00	0.000000	0.000000	0.000000	0.000000	0.566474	0.566668	0.022918
130.00	0.000000	0.000000	0.000000	0.000000	0.564926	0.565137	0.022872
140.00	0.000000	0.000000	0.000000	0.000000	0.563270	0.563492	0.022822
150.00	0.000000	0.000000	0.000000	0.000000	0.561556	0.561782	0.022771
175.00	0.000000	0.000000	0.000000	0.000000	0.557390	0.557596	0.022646
200.00	0.000000	0.000000	0.000000	0.000000	0.553983	0.554136	0.022544

Table 13.

Wavelength, λ = 691.98 nm.

z, m	E_d^R	E_u^R	E_d^{CY}	E_u^{CY}	αk_d, m^{-1}	αk_u, m^{-1}	$R(z)$, %
0.01	0.000018	0.000959	0.000876	0.053145	0.572821	0.180944	0.023147
0.10	0.000174	0.000933	0.008518	0.052822	0.572753	0.176946	0.023147
0.20	0.000334	0.000905	0.016523	0.052465	0.572674	0.172618	0.023147
0.30	0.000480	0.000878	0.024042	0.052112	0.572590	0.168411	0.023147
0.50	0.000734	0.000826	0.037730	0.051416	0.572411	0.160358	0.023147
0.70	0.000944	0.000778	0.049775	0.050731	0.572212	0.152785	0.023147
1.00	0.001187	0.000711	0.065142	0.049726	0.571875	0.142305	0.023146
2.00	0.001563	0.000534	0.098532	0.046554	0.570306	0.114415	0.023146
3.00	0.001561	0.000408	0.113929	0.043628	0.567724	0.095539	0.023146
5.00	0.001202	0.000253	0.118980	0.038398	0.556597	0.075270	0.023145
7.00	0.000834	0.000172	0.111006	0.033864	0.528131	0.067032	0.023144
10.00	0.000491	0.000113	0.094351	0.028112	0.413211	0.062782	0.023142
15.00	0.000266	0.000074	0.069756	0.020691	0.135291	0.061055	0.023140
20.00	0.000185	0.000055	0.051445	0.015274	0.067291	0.060446	0.023137
25.00	0.000140	0.000043	0.038019	0.011301	0.060742	0.059996	0.023133
30.00	0.000109	0.000033	0.028159	0.008379	0.059841	0.059582	0.023130
35.00	0.000085	0.000026	0.020898	0.006226	0.059396	0.059179	0.023125
40.00	0.000067	0.000020	0.015541	0.004635	0.058993	0.058781	0.023120
50.00	0.000041	0.000013	0.008646	0.002584	0.058202	0.057994	0.023109
60.00	0.000025	0.000008	0.004848	0.001452	0.057424	0.057220	0.023094
70.00	0.000016	0.000005	0.002739	0.000822	0.056663	0.056466	0.023076
80.00	0.000010	0.000003	0.001559	0.000469	0.055928	0.055738	0.023054
90.00	0.000006	0.000002	0.000894	0.000269	0.055224	0.055044	0.023027
100.00	0.000004	0.000001	0.000516	0.000156	0.054558	0.054389	0.022996
110.00	0.000002	0.000001	0.000300	0.000091	0.053934	0.053777	0.022959
120.00	0.000001	0.000000	0.000175	0.000053	0.053355	0.053209	0.022917
130.00	0.000001	0.000000	0.000103	0.000031	0.052822	0.052688	0.022870
140.00	0.000001	0.000000	0.000061	0.000018	0.052334	0.052213	0.022821
150.00	0.000000	0.000000	0.000036	0.000011	0.051891	0.051781	0.022769
175.00	0.000000	0.000000	0.000010	0.000003	0.050964	0.050880	0.022645
200.00	0.000000	0.000000	0.000003	0.000001	0.050257	0.050193	0.022543

Wavelength Dependent Data for 10 m Depth.

Table 14.

Depth, Z = 10.00 m.

z, m	E_d^q	E_d^s	E_d^E	E_u^s	$\alpha^E k_d^E$, *m-1*	$\alpha^E k_u^E$, *m-1*	$R_s^E(z)$, %
399.8868	36.326625	1.322398	37.649023	0.539003	0.071092	0.070792	1.431651
401.9064	37.466563	1.337030	38.803593	0.549537	0.070413	0.070120	1.416201
403.9466	38.509388	1.347023	39.856411	0.557926	0.069787	0.069502	1.399840
406.0075	39.543571	1.355610	40.899181	0.565100	0.069257	0.068980	1.381691
408.0896	40.628549	1.364969	41.993518	0.573165	0.068672	0.068402	1.364888
410.1932	41.652455	1.371249	43.023703	0.579620	0.068140	0.067877	1.347210
412.3185	41.963307	1.353347	43.316653	0.573271	0.067920	0.067665	1.323443
414.4660	42.241470	1.334404	43.575874	0.565762	0.067785	0.067538	1.298338
416.6360	42.512618	1.315368	43.827987	0.558310	0.067637	0.067398	1.273867
418.8288	42.759097	1.295670	44.054767	0.550147	0.067541	0.067310	1.248780
421.0448	42.720339	1.267874	43.988212	0.540708	0.067173	0.066949	1.229212
423.2844	42.318922	1.230208	43.549130	0.528856	0.066565	0.066346	1.214389
425.5480	41.903003	1.193053	43.096056	0.517153	0.065943	0.065731	1.200002
427.8359	41.434024	1.155298	42.589321	0.504616	0.065373	0.065167	1.184842
430.1485	41.090592	1.121877	42.212468	0.493179	0.064886	0.064686	1.168324
432.4863	42.912011	1.146499	44.058510	0.499737	0.065378	0.065186	1.134257
434.8496	44.715494	1.168988	45.884482	0.504869	0.065921	0.065737	1.100305
437.2389	46.448474	1.188018	47.636492	0.507217	0.066623	0.066447	1.064765
439.6545	48.222363	1.206697	49.429060	0.510031	0.067233	0.067065	1.031844
442.0971	50.292944	1.232254	51.525198	0.529622	0.066049	0.065884	1.027889
444.5669	52.481402	1.259133	53.740535	0.553095	0.064557	0.064396	1.029195
447.0644	54.751762	1.286171	56.037933	0.577576	0.063083	0.062925	1.030688
449.5902	57.090438	1.312985	58.403423	0.602944	0.061626	0.061471	1.032379
452.1447	58.284583	1.312229	59.596813	0.616511	0.060175	0.060023	1.034470
454.7284	59.212164	1.304880	60.517044	0.626609	0.058817	0.058667	1.035425
457.3418	60.164726	1.297699	61.462424	0.637461	0.057442	0.057294	1.037155
459.9854	61.143236	1.290690	62.433926	0.649136	0.056050	0.055906	1.039718
462.6597	61.835887	1.277660	63.113548	0.662822	0.054287	0.054144	1.050206
465.3653	62.539380	1.264755	63.804135	0.677731	0.052506	0.052365	1.062205
468.1028	63.258473	1.252042	64.510515	0.693604	0.050741	0.050601	1.075180
470.8726	63.828698	1.235917	65.064615	0.701239	0.049498	0.049361	1.077758
473.6754	64.091638	1.213100	65.304738	0.689528	0.049371	0.049239	1.055862
476.5118	64.298840	1.189489	65.488329	0.676095	0.049334	0.049207	1.032391
479.3824	64.498435	1.166085	65.664520	0.662592	0.049311	0.049189	1.009056
482.2877	64.229435	1.135348	65.364783	0.656353	0.048429	0.048310	1.004138
485.2285	63.877437	1.104112	64.981549	0.654470	0.047194	0.047077	1.007164
488.2053	63.402710	1.071481	64.474191	0.650419	0.046048	0.045933	1.008805
491.2190	63.218887	1.044232	64.263118	0.644415	0.045260	0.045149	1.002776
494.2700	63.496659	1.024663	64.521322	0.635179	0.045026	0.044918	0.984449
497.3592	63.788773	1.005592	64.794365	0.626451	0.044772	0.044668	0.966829
500.4872	63.770981	0.981764	64.752745	0.609333	0.044908	0.044809	0.941015
503.6549	62.070610	0.932023	63.002634	0.550229	0.047182	0.047091	0.873343
506.8629	60.422414	0.885015	61.307429	0.498303	0.049436	0.049352	0.812794

continued

Table 14 (continuation).

Depth, Z = 10.00 m.

z, m	E_d^q	E_d^s	E_d^E	E_u^s	$\alpha^E k_d^E$, *m-1*	$\alpha^E k_u^E$, *m-1*	$R_s^E(z)$, %
510.1120	58.797143	0.840151	59.637294	0.451965	0.051701	0.051624	0.757856
513.4030	56.294665	0.784456	57.079121	0.395156	0.055164	0.055094	0.692296
516.7368	53.898483	0.732587	54.631069	0.347013	0.058605	0.058542	0.635193
520.1142	51.658685	0.684973	52.343657	0.306456	0.061982	0.061925	0.585470
523.5360	51.870600	0.671409	52.542010	0.297592	0.062512	0.062459	0.566389
527.0031	52.138242	0.658768	52.797011	0.289591	0.062977	0.062927	0.548500
530.5165	52.144612	0.643050	52.787662	0.279373	0.063669	0.063622	0.529239
534.0770	50.831864	0.611712	51.443576	0.259612	0.065115	0.065071	0.504654
537.6856	49.558127	0.581944	50.140071	0.241347	0.066571	0.066530	0.481345
541.3433	48.205182	0.552289	48.757471	0.222015	0.068607	0.068570	0.455346
545.0512	46.839805	0.523545	47.363350	0.202091	0.071365	0.071332	0.426681
548.8101	45.458587	0.495700	45.954287	0.183710	0.074240	0.074210	0.399766
552.6213	43.764102	0.465572	44.229674	0.166144	0.077004	0.076978	0.375640
556.4858	41.969741	0.435577	42.405318	0.149992	0.079703	0.079679	0.353711
560.4047	40.147042	0.406473	40.553516	0.134845	0.082625	0.082604	0.332511
564.3792	38.149011	0.376798	38.525808	0.119489	0.086310	0.086291	0.310154
568.4105	36.167489	0.348491	36.515980	0.105693	0.090111	0.090095	0.289442
572.4998	32.841719	0.308799	33.150518	0.084989	0.099049	0.099037	0.256372
576.6483	28.899953	0.265313	29.165267	0.064564	0.111648	0.111640	0.221373
580.8575	24.768588	0.222136	24.990724	0.047624	0.126236	0.126232	0.190567
585.1285	19.168591	0.168194	19.336786	0.030457	0.148600	0.148601	0.157509
589.4628	14.759739	0.126788	14.886527	0.019784	0.171402	0.171406	0.132899
593.8617	10.017517	0.084549	10.102066	0.010646	0.210264	0.210273	0.105386
598.3269	6.648741	0.055217	6.703958	0.005738	0.251934	0.251946	0.085586
602.8596	4.957238	0.040422	4.997660	0.003728	0.281434	0.281448	0.074592
607.4616	3.963447	0.031683	3.995130	0.002690	0.303613	0.303629	0.067331
612.1344	3.331260	0.026075	3.357335	0.002086	0.320412	0.320429	0.062134
616.8796	2.980191	0.022803	3.002993	0.001762	0.330537	0.330553	0.058661
621.6990	2.697076	0.020164	2.717239	0.001513	0.339040	0.339057	0.055697
626.5943	2.489859	0.018173	2.508033	0.001339	0.344543	0.344560	0.053376
631.5672	2.316663	0.016507	2.333170	0.001194	0.350061	0.350078	0.051157
636.6198	2.202820	0.015321	2.218141	0.001088	0.355358	0.355376	0.049071
641.7538	2.036207	0.013830	2.050036	0.000960	0.362643	0.362661	0.046816
646.9713	1.782888	0.011836	1.794724	0.000794	0.373606	0.373624	0.044238
652.2744	1.449355	0.009426	1.458781	0.000597	0.392956	0.392975	0.040937
657.6651	1.068866	0.006832	1.075698	0.000398	0.423549	0.423568	0.036959
663.1456	0.830479	0.005210	0.835690	0.000283	0.449631	0.449650	0.033880
668.7182	0.670369	0.004125	0.674494	0.000212	0.472517	0.472537	0.031373
674.3854	0.588341	0.003539	0.591880	0.000177	0.483770	0.483790	0.029821
680.1493	0.525224	0.003085	0.528309	0.000151	0.492139	0.492159	0.028526
686.0127	0.364339	0.002103	0.366443	0.000096	0.522051	0.522071	0.026163
691.9780	0.207678	0.001186	0.208865	0.000048	0.574020	0.574042	0.023144
698.0480	0.079826	0.000458	0.080284	0.000015	0.671245	0.671268	0.019246

Table 15.

Depth, Z = 10.00 m.

z, m	E_d^R	E_u^R	E_d^{CY}	E_u^{CY}	αk_d, m^{-1}	αk_u, m^{-1}	$R(z)$, %
399.8868	0.050267	0.018194	0.023994	0.006251	0.071366	0.072151	1.422697
401.9064	0.050401	0.018545	0.030227	0.007880	0.070670	0.071567	1.407452
403.9466	0.050656	0.018928	0.037325	0.009734	0.070026	0.071059	1.391306
406.0075	0.050973	0.019325	0.045163	0.011779	0.069478	0.070662	1.373391
408.0896	0.051843	0.019954	0.053597	0.013983	0.068873	0.070224	1.356801
410.1932	0.053515	0.020903	0.062345	0.016268	0.068319	0.069849	1.339344
412.3185	0.055992	0.022150	0.070977	0.018499	0.068073	0.069801	1.315865
414.4660	0.059153	0.023675	0.079186	0.020608	0.067910	0.069829	1.291057
416.6360	0.062698	0.025375	0.086626	0.022512	0.067736	0.069830	1.266869
418.8288	0.065908	0.026932	0.092887	0.024100	0.067616	0.069859	1.242067
421.0448	0.068488	0.028261	0.097806	0.025366	0.067226	0.069623	1.222719
423.2844	0.070280	0.029285	0.101109	0.026239	0.066598	0.069154	1.208064
425.5480	0.071323	0.029982	0.102490	0.026617	0.065961	0.068642	1.193838
427.8359	0.071846	0.030439	0.101832	0.026460	0.065380	0.068142	1.178846
430.1485	0.071930	0.030675	0.099154	0.025770	0.064887	0.067671	1.162506
432.4863	0.071289	0.030384	0.094136	0.024366	0.065393	0.067943	1.128783
434.8496	0.070308	0.029913	0.087602	0.022578	0.065950	0.068244	1.095163
437.2389	0.069020	0.029263	0.079859	0.020483	0.066666	0.068682	1.059959
439.6545	0.067557	0.028553	0.071417	0.018237	0.067292	0.069029	1.027339
442.0971	0.066489	0.028342	0.063253	0.016204	0.066124	0.067685	1.023430
444.5669	0.065273	0.028145	0.055023	0.014161	0.064648	0.066041	1.024735
447.0644	0.063817	0.027863	0.046924	0.012135	0.063188	0.064409	1.026227
449.5902	0.062224	0.027528	0.039233	0.010196	0.061743	0.062797	1.027915
452.1447	0.060999	0.027332	0.032160	0.008400	0.060299	0.061227	1.030000
454.7284	0.060407	0.027356	0.025835	0.006781	0.058944	0.059769	1.030960
457.3418	0.060571	0.027671	0.020352	0.005369	0.057571	0.058324	1.032688
459.9854	0.061346	0.028226	0.015719	0.004169	0.056180	0.056890	1.035241
462.6597	0.062565	0.029114	0.011930	0.003189	0.054415	0.055118	1.045653
465.3653	0.063880	0.030182	0.008879	0.002392	0.052631	0.053334	1.057562
468.1028	0.065162	0.031413	0.006478	0.001760	0.050863	0.051563	1.070438
470.8726	0.066196	0.032562	0.004619	0.001262	0.049615	0.050295	1.073008
473.6754	0.066607	0.033187	0.003209	0.000876	0.049484	0.050103	1.051316
476.5118	0.066638	0.033635	0.002185	0.000596	0.049442	0.049999	1.028057
479.3824	0.066277	0.033888	0.001459	0.000397	0.049415	0.049908	1.004928
482.2877	0.065891	0.034405	0.000960	0.000262	0.048528	0.049000	1.000064
485.2285	0.065373	0.035002	0.000619	0.000170	0.047288	0.047755	1.003078
488.2053	0.064650	0.035510	0.000393	0.000109	0.046137	0.046594	1.004721
491.2190	0.063690	0.035802	0.000244	0.000068	0.045346	0.045771	0.998754
494.2700	0.062482	0.035745	0.000147	0.000041	0.045108	0.045472	0.980586
497.3592	0.061269	0.035604	0.000087	0.000024	0.044851	0.045164	0.963116
500.4872	0.059945	0.035108	0.000052	0.000014	0.044985	0.045246	0.937509
503.6549	0.057990	0.033317	0.000029	0.000008	0.047253	0.047378	0.870331
506.8629	0.056004	0.031539	0.000016	0.000004	0.049502	0.049486	0.810194

continued

Table 15 (continuation).

Depth, Z = 10.00 m.

z, m	E_d^R	E_u^R	E_d^{CY}	E_u^{CY}	αk_d, m^{-1}	αk_u, m^{-1}	$R(z)$, %
510.1120	0.053913	0.029800	0.000008	0.000002	0.051762	0.051590	0.755602
513.4030	0.051391	0.027646	0.000004	0.000001	0.055219	0.054786	0.690420
516.7368	0.048854	0.025669	0.000002	0.000000	0.058655	0.057922	0.633620
520.1142	0.046427	0.023879	0.000002	0.000000	0.062027	0.060968	0.584138
523.5360	0.044832	0.023082	0.000000	0.000000	0.062556	0.061419	0.565147
527.0031	0.043392	0.022330	0.000000	0.000000	0.063021	0.061828	0.547340
530.5165	0.041923	0.021432	0.000000	0.000000	0.063711	0.062479	0.528164
534.0770	0.040232	0.020213	0.000000	0.000000	0.065155	0.063848	0.503680
537.6856	0.038444	0.018887	0.000000	0.000000	0.066608	0.065277	0.480463
541.3433	0.036454	0.017360	0.000000	0.000000	0.068643	0.067296	0.454560
545.0512	0.034299	0.015706	0.000000	0.000000	0.071400	0.070026	0.425994
548.8101	0.032148	0.014123	0.000000	0.000000	0.074274	0.072898	0.399166
552.6213	0.030193	0.012758	0.000000	0.000000	0.077037	0.075653	0.375112
556.4858	0.028516	0.011645	0.000000	0.000000	0.079733	0.078304	0.353245
560.4047	0.027092	0.010743	0.000000	0.000000	0.082652	0.081100	0.332102
564.3792	0.025743	0.009923	0.000000	0.000000	0.086333	0.084535	0.309800
568.4105	0.024449	0.009184	0.000000	0.000000	0.090130	0.088036	0.289135
572.4998	0.022613	0.008120	0.000000	0.000000	0.099060	0.096101	0.256133
576.6483	0.020507	0.006985	0.000000	0.000000	0.111649	0.107247	0.221196
580.8575	0.018410	0.005957	0.000000	0.000000	0.126225	0.119781	0.190436
585.1285	0.015982	0.004883	0.000000	0.000000	0.148564	0.137739	0.157419
589.4628	0.013866	0.004054	0.000000	0.000000	0.171335	0.154485	0.132836
593.8617	0.011318	0.003169	0.000000	0.000000	0.210136	0.178987	0.105347
598.3269	0.009207	0.002514	0.000000	0.000000	0.251716	0.198329	0.085560
602.8596	0.007848	0.002115	0.000000	0.000000	0.281138	0.207873	0.074573
607.4616	0.006839	0.001821	0.000000	0.000000	0.303256	0.213731	0.067315
612.1344	0.006025	0.001580	0.000000	0.000000	0.320013	0.218821	0.062120
616.8796	0.005363	0.001377	0.000000	0.000000	0.330130	0.224883	0.058649
621.6990	0.004715	0.001176	0.000000	0.000000	0.338643	0.232755	0.055686
626.5943	0.004102	0.000988	0.000000	0.000000	0.344175	0.242134	0.053366
631.5672	0.003490	0.000806	0.000000	0.000000	0.349735	0.253776	0.051149
636.6198	0.002913	0.000643	0.000001	0.000000	0.355085	0.267487	0.049062
641.7538	0.002389	0.000504	0.000008	0.000002	0.362409	0.280425	0.046809
646.9713	0.001938	0.000394	0.000084	0.000024	0.373378	0.287653	0.044231
652.2744	0.001552	0.000310	0.000603	0.000175	0.392595	0.263737	0.040931
657.6651	0.001227	0.000247	0.003098	0.000901	0.422217	0.165106	0.036955
663.1456	0.001010	0.000207	0.011818	0.003450	0.443801	0.092763	0.033877
668.7182	0.000859	0.000180	0.033360	0.009774	0.452493	0.071246	0.031370
674.3854	0.000767	0.000163	0.070886	0.020801	0.437621	0.065622	0.029818
680.1493	0.000697	0.000149	0.111078	0.032628	0.416017	0.063888	0.028524
686.0127	0.000601	0.000132	0.121955	0.036008	0.405583	0.063095	0.026160
691.9780	0.000491	0.000113	0.094351	0.028112	0.413211	0.062782	0.023142
698.0480	0.000367	0.000090	0.050021	0.015140	0.435982	0.062642	0.019245

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Appendix I: Simplified Estimation of Limiting Values of the Spectral Curve for the Irradiance Attenuation Coefficient

Here we present a highly simplified estimate for the Irradiance Attenuation Coefficient (IAC) equation which can clarify an abnormal spectral behavior of IAC displayed in Figures 32, 35, 37, 40, 43, 45, 48, 51, 53, 56, 59 and 61.

Let λ_R be a wavelength in the red region of the spectrum and neglect the spectral dependence of natural light, *i. e.* consider its spectral dependence as uniform. Then a simplified expression for irradiances can be expressed as

$$E_i(\lambda_R, z) = E_i^0 \left\{ e^{-k_i^E(\lambda_R)z} + b_R \left(\frac{400}{\lambda_R} \right)^4 e^{-\left\langle k_i^E \right\rangle_R^R z} + C\eta_C \left\langle a_C^0 \right\rangle_R^C e^{-\left\langle k_i^E \right\rangle_R^C z} \right\}, \tag{1A}$$

where $i = d, u$; E_i^0 is an irradiance just under the sea surface; k_i^E is an elastic IAC; b_R is a Raman scattering coefficient; η_C is a chlorophyll fluorescence quantum efficiency; angular brackets denote averaging over Raman scattering and fluorescence excitation bands

$$\langle \Phi \rangle_R^R \equiv \int_{-\sigma_\lambda^R/2}^{\sigma_\lambda^R/2} \Phi(\lambda_R - \Delta\lambda^R + \lambda') \frac{d\lambda'}{\sigma_\lambda^R}, \tag{2A}$$

$$\langle \Phi \rangle_R^C \equiv \int_{400}^{685} \Phi(\lambda') \frac{d\lambda'}{\sigma_\lambda^C}, \tag{3A}$$

where $\Delta\lambda^R$ is the shift of the Raman scattering band, σ_λ^R is its width; σ_λ^C is the width of the chlorophyll fluorescence band.

The inelastic IAC $k_i(\lambda_R, z) \equiv -\frac{d}{dz} \log E_i(\lambda_R, z)$ will be

$$k_i(\lambda_R, z) = \frac{k_i^E(\lambda_R) e^{-k_i^E(\lambda_R)z} + \left\langle k_i^E \right\rangle_R^R b_R \left(\frac{400}{\lambda_R} \right)^4 + \left\langle k_i^E \right\rangle_R^C C\eta_C \left\langle a_C^0 \right\rangle_R^C}{e^{-k_i^E(\lambda_R)z} + b_R \left(\frac{400}{\lambda_R} \right)^4 + C\eta_C \left\langle a_C^0 \right\rangle_R^C}, \tag{4A}$$

where we have taken into account that $k_i^E(\lambda_R) >> \left\langle k_i^E \right\rangle_R^R$ and $k_i^E(\lambda_R) >> \left\langle k_i^E \right\rangle_R^C$.

Let us consider only Raman scattering, *i. e.* $C = 0$:

$$k_i(\lambda_R, z) = \frac{k_i^E(\lambda_R) e^{-k_i^E(\lambda_R)z} + \left\langle k_i^E \right\rangle_R^R b_R \left(\frac{400}{\lambda_R} \right)^4}{e^{-k_i^E(\lambda_R)z} + b_R \left(\frac{400}{\lambda_R} \right)^4}, \tag{5A}$$

near the sea surface both the second terms in the numerator and the denominator of Eqn. (5A) are much smaller in comparison with $k_i^E(\lambda_R)$ and unity, and $k_i(\lambda_R, z) \approx k_i^E(\lambda_R)$. With increase of depth

z both the first terms in the numerator and the denominator of Eqn. (5A) exponentially diminish and at

$$z \geq \frac{1}{k_i^E(\lambda_R)} \log \frac{k_i^E(\lambda_R)}{\left\langle k_i^E \right\rangle_R^R b_R \left(\frac{400}{\lambda_R} \right)^4}, \qquad (6A)$$

inelastic IAC $k_i(\lambda_R, z)$ asymptotically approaches the averaged (over shorter wavelengths) elastic IAC $\left\langle k_i^E \right\rangle_R^R$. This leads to a *widening* of the transparency window and a *flattening* of the spectral curves for IAC with increasing depth. The consideration of the chlorophyll fluorescence when $C \neq 0$ only enhances this effect in the vicinity of the fluorescence emission band near 685 *nm*.

1997 addition:

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Some later modeling results are published in:

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